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Armington Elasticities for Belgium

Estimates of the Elasticity of Substitution between Products from Different Origins

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Abstract – With the goal of calibrating macroeconomic models for Belgium, this paper estimates the elasticity of substitution between products originating from different countries. Following Feenstra (1994) and Soderbery (2015), the elasticity is obtained by applying a system of equations estimator. Armed with the BACI dataset, it estimates the elasticity for different levels of (dis)aggregation and provides evidence of a strong heterogeneity. At a highly disaggregated level covering about 4,500 HS products, the elasticity ranges from 1.12 to 70.69 with a median of 4.86. Across CPA product groups, the value of the elasticity ranges from 2.40 to 16.63. These results can be used to calibrate a multi-sector macroeconomic model. Accounting for the heterogeneity across CPA products, the aggregate elasticity for a one-sector model ranges from 1.98 to 2.48. The results also support the presence of measurement errors in the data no matter the (dis)aggregation level considered.

JEL Classification – C13, C23, F14, F32, F41.
Keywords – Armington Elasticity, Import Elasticity, Calibration of Macroeconomic Models, Aggregation, Sectoral Estimates, Transmission of Shocks.

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Executive summary

The Armington elasticity, or the response of import flows to a shock to international prices plays a crucial role in open macroeconomic and international trade models, especially for a small open economy such as Belgium. In fact, the predictions of those models depend heavily on the value of the parameter. And yet, the value of the Armington elasticity is still highly debated and empirical studies offer a range of values that is too large to help settling the issue. Moreover, the values taken by this parameter to replicate business cycles features in macroeconomic models tend to be too low compared to those from micro-econometric studies. As a result, it is challenging to calibrate macroeconomic models by relying on the existing literature.

The goal of this study is to estimate Armington elasticities that could be used to calibrate a one-sector as well as a multi-sector open macroeconomic models of the Belgian economy. To this end, it first estimates the elasticity across the highly disaggregated 6-digit products of the harmonized system (HS). It then aggregates the data for classification of products by activity (CPA) product groups and performs estimations at that level as well. The resulting estimates can be used to calibrate a multi-sector model of the Belgian economy such as DynEMItE[1]. For a one sector-model such as the Belgian version of QUEST III R&D[2], the elasticities estimated across CPA product groups are aggregated to obtain a unique aggregate elasticity.

The estimation of the Armington elasticity is confronted with at least two endogeneity issues. The first is the simultaneity bias which comes from the simultaneous determination of both prices and quantities. The second is due to measurement errors as prices are not observable in trade datasets and are proxied with unit values. To tackle these issues, we use the framework developed by Feenstra (1994). The latter deals with endogeneity by estimating a simultaneous system of equations containing both the supply and the demand of imports. In addition, it makes use of the panel structure of trade datasets to construct internal instruments.

The methodology is applied to data from the BACI dataset to estimate Armington elasticities for different levels of (dis)aggregation. Results point to a strong heterogeneity in the estimates, not only across HS 6-digit products, but also across CPA product groups. For the former, the estimates range from 1.12 to 70.69. For the latter, the estimated elasticities take values between 2.40 to 16.63. The results suggest that the calibration of macroeconomic models should account for this heterogeneity across product groups. They also suggest that in order to limit the effect of heterogeneity bias in a one-sector model, it is desirable to perform estimates with disaggregated data before aggregating them properly. Doing so implies an aggregate Armington elasticity that lies between 1.98 and 2.48 for a one-sector model.

1 See Verwerft (2022)

2 See Roeger, Varga, and in 't Veld (2008)

Synthèse

L'élasticité d'Armington, ou la réaction des flux d'importations à la suite d'un choc sur les prix internationaux, joue un rôle crucial dans les modèles macroéconomiques ouverts et ceux du commerce international, en particulier pour une petite économie ouverte comme la Belgique. En effet, les prédictions de ces modèles dépendent fortement de la valeur de ce paramètre. Or, la valeur de l'élasticité d'Armington est encore très débattue et les études empiriques offrent une fourchette de valeurs trop large pour permettre de trancher la question. De plus, les valeurs prises par ce paramètre pour reproduire les caractéristiques des cycles économiques dans les modèles macroéconomiques tendent à être trop faibles par rapport à celles obtenues par les études micro-économétriques. Par conséquent, il est difficile de calibrer les modèles macroéconomiques en s'appuyant sur la littérature existante.

L'objectif de cette étude est d'estimer les élasticités d'Armington de façon à calibrer un modèle macroéconomique ouvert à un ou à plusieurs secteurs pour la Belgique. À cette fin, elle estime d'abord l'élasticité au niveau très désagrégé des produits à 6 chiffres de la nomenclature du système harmonisé (SH). Il agrège ensuite les données pour les groupes de produits de la classification des produits par activité (CPA) et effectue des estimations à ce niveau également. Les estimations qui en résultent peuvent être utilisées pour calibrer un modèle multisectoriel de l'économie belge tel que DynEMItE. Pour un modèle à un secteur tel que la version belge de QUEST III R&D, les élasticités estimées obtenues pour les groupes de produits CPA sont agrégées pour obtenir une élasticité agrégée unique.

L'estimation de l'élasticité d'Armington est confrontée à au moins deux problèmes d'endogénéité. Le premier est le biais de simultanéité qui provient de la détermination simultanée des prix et des quantités. Le second est dû aux erreurs de mesure, car les prix ne sont pas observables dans les bases de données du commerce international et sont remplacés par des valeurs unitaires. Pour résoudre ces problèmes, nous utilisons le cadre développé par Feenstra (1994). Ce dernier traite l'endogénéité en estimant un système d'équations simultanées contenant à la fois l'offre et la demande d'importations. En outre, il exploite la structure de panel des bases de données du commerce international pour construire des instruments internes.

La méthodologie est appliquée aux données de la base de données BACI afin d'estimer les élasticités d'Armington pour différents niveaux d'(de) (dés)agrégation. Les résultats indiquent une forte hétérogénéité dans les estimations, non seulement entre les produits à 6 chiffres du SH, mais aussi entre les groupes de produits CPA. Pour les premiers, les estimations vont de 1,12 à 70,69. Pour les seconds, les élasticités estimées se situent entre 2,40 et 16,63. Les résultats suggèrent que la calibration des modèles macroéconomiques devrait tenir compte de cette hétérogénéité entre les groupes de produits. Ils suggèrent également que pour limiter l'effet du biais d'hétérogénéité dans un modèle à un secteur, il est souhaitable d'effectuer des estimations avec des données désagrégées avant de les agréger correctement. Cela implique une élasticité d'Armington agrégée qui se situe entre 1,98 et 2,48 pour un modèle à un secteur.

Synthese

De Armington-elasticiteit, of de reactie van invoerstromen op een schok op de internationale prijzen, speelt een cruciale rol in open macro-economische modellen en internationale handelsmodellen, vooral voor een kleine open economie als België. De voorspellingen van deze modellen zijn sterk afhankelijk van de waarde van deze parameter. Er bestaat echter geen consensus over de waarde van de Armington-elasticiteit en de empirische studies bieden een te brede waaier van waarden om de vraag te kunnen beantwoorden. Bovendien zijn de waarden die deze parameter aanneemt om de kenmerken van economische cycli in macro-economische modellen te reproduceren vaak te laag in vergelijking met de waarden die door micro-econometrische studies worden verkregen. Daardoor is het moeilijk om macro-economische modellen te kalibreren op basis van bestaande literatuur.

Het doel van deze studie is de Armington-elasticiteit te schatten om een open macro-economisch één-sector- of multisectormodel te kalibreren van de Belgische economie. Hiertoe wordt eerst de elasticiteit geschat op het zeer gedesaggregeerde niveau van de 6-cijferige producten van de nomenclatuur van het geharmoniseerd systeem (GS). Vervolgens worden de gegevens voor de productgroepen in de classificatie van producten gekoppeld aan activiteiten (CPA) samengevoegd en worden ook op dit niveau schattingen gemaakt. De resulterende schattingen kunnen worden gebruikt om een multisectormodel van de Belgische economie zoals DynEMItE te kalibreren. Voor een één-sectormodel zoals de Belgische versie van QUEST III R&D, worden de elasticiteiten geschat over de CPA-productgroepen samengevoegd om één geaggregeerde elasticiteit te verkrijgen.

De schatting van de Armington-elasticiteit wordt geconfronteerd met ten minste twee endogeniteitsproblemen. De eerste is de 'simultaneity bias', die ontstaat door de gelijktijdige bepaling van prijzen en hoeveelheden. De tweede is te wijten aan meetfouten, aangezien prijzen niet waarneembaar zijn in datasets over internationale handel en worden vervangen door eenheidswaarden. Om deze problemen op te lossen, gebruiken we het door Feenstra (1994) ontwikkelde kader. Dit laatste gaat om met de endogeniteit door een systeem van gelijktijdige vergelijkingen te schatten dat zowel het invoeraanbod als de invoervraag bevat. Daarnaast maakt het gebruik van de panelstructuur van datasets over internationale handel om interne instrumenten te bouwen.

De methodologie wordt toegepast op gegevens uit de BACI-databank om de Armington-elasticiteit te schatten voor verschillende (des)aggregatieniveaus. De resultaten laten een hoge mate van heterogeniteit in de schattingen zien, niet alleen tussen de 6-cijferige producten van het GS, maar ook tussen de CPA-productgroepen. Schattingen voor de eerste variëren van 1,12 tot 70,69. Voor de tweede liggen de geschatte elasticiteiten tussen 2,40 en 16,63. De resultaten suggereren dat de kalibratie van macro-economische modellen rekening moet houden met deze heterogeniteit tussen productgroepen. Ze suggereren ook dat het, om het effect van heterogeniteitsvertekening in een één-sectormodel te beperken, aangewezen is om schattingen uit te voeren met gedesaggregeerde gegevens alvorens ze naar behoren te aggregeren. Dit impliceert een geaggregeerde Armington-elasticiteit tussen 1,98 en 2,48 voor een één-sectormodel.

1. Introduction

The elasticity of substitution between products from different countries, also called the Armington elasticity, measures the reaction of relative demands to changes in relative prices. It is relevant for several applications in open economy models. For instance, because it measures the willingness of the consumer to substitute domestic products for foreign ones, it influences how a country's trade balance adjusts to a shock (Backus, Kehoe, and Kydland, 1994). It also reflects the competitive position of an economy, especially for small open economies. In addition, it is called upon in the quantification of the welfare effects of trade policies including globalization and the gains from varieties as in Broda and Weinstein (2006) or Caliendo and Parro (2015).

Unsurprisingly, with such a crucial role, the estimation of the Armington elasticity has attracted much attention in the literature. However, two issues plague its use when it comes to calibrated models. First, the range of the estimates produced by the literature is quite wide. This is well illustrated by the meta-study of Bajzik et al. (2020) which provides values of the elasticity that range from 0 to 8. As possible causes of this dispersion, the authors pointed out data characteristics (aggregation, frequency, data types), industry characteristics, as well as estimation techniques. In any case, the uncertainty around the size of the elasticity makes it difficult to use a unique value in calibrated models.

The second issue relates to the discrepancy between estimated values which come typically from microeconometric studies, and the values commonly used in standard open economy macro-models. Despite their dispersion, microeconomic estimates of the Armington elasticity are centered around 3. But to replicate business cycle features such as the volatility of terms of trade or the negative correlation between terms of trade (defined here as the ratio of import price to export price) and trade balance found in data, a value of 1.5 or lower is typically assumed in macroeconomic models (Ruhl, 2008). Moreover, estimations within the same class of models sometimes result in values slightly lower than 1 (Heathcote and Perri (2002)).

Yet, the value assigned to this parameter can influence the results of macroeconomic models quantitatively, or even qualitatively. This is reflected by the sensitivity analysis of Schürenberg-Frosch (2015) who focused on 50 CGE models and found conflicting results depending on the value of the Armington elasticity. The Belgian version of QUEST III R&D, the model of the European Commission used to evaluate the quantitative effects of structural reforms, is not immune to this issue neither.

To illustrate this point, we stimulate a 0.5% permanent shock to public investment for two different values of the elasticity, respectively 3 and 2. The results are shown in Table 9 in the Appendix. A positive public investment shock directly increases the aggregate demand and expands the supply side of the economy through the public capital stock. In the long run, real GDP, employment rate, labour productivity, private investment, private consumption, imports, and exports all increase while GDP deflator decreases. This holds for both values of the Armington elasticity. Moreover, the effects are quantitatively comparable for both values. However, in the short run, where the demand side of the public investment shock prevails, the results depend on the value of the Armington elasticity. This is particularly true for demand components such as private consumption, private investment and net exports.

Following the shock, GDP deflator increases in the short run, leading to a decrease in import prices. As a consequence, imports increase. The higher the Armington elasticity, the higher the reaction of imports which allows more consumption and a limited crowding-out of investment. Hence, changing the value of that elasticity from 3 to 2 in the model has significant quantitative implications on macroeconomic aggregates such as private consumption, private investment and trade flows in the short run.

This working paper has two goals. The first is to estimate and characterize the Armington elasticities at a highly disaggregated but also at a less disaggregated level. The second is to make use of these estimates to calibrate a macroeconomic model for the Belgian economy. This is useful for a one sector model such as the Belgian version of QUEST III R&D, but also for a multi-sector model such as DynEMItE, a model under development at the Belgian Federal Planning Bureau.

To estimate Armington elasticities, we rely on the methodology developed by Feenstra (1994). It consists of a structural estimator combining an import demand and an import supply equation. Combined with the panel structure of the data, this allows to tackle endogeneity issues without resorting to valid external instruments. The estimation is first performed on the BACI database[3], a database of yearly information on bilateral trade flows in the universe of 6-digit level of the harmonized system (HS6). We find strong heterogeneity in the estimates of Armington elasticity across the 6-digit HS products. The value of the estimates varies from a minimum of 1.12 to a maximum of 70.69. The median value of the product-level elasticities is 4.86 while the unweighted mean is 6.34.

Then, the data are aggregated at the level of CPA (Classification of Products by Activity) A64 product groups and the estimation is performed at that level. The heterogeneity in the elasticities is also observed across CPA product groups, but to a lesser extent. The estimates range from 2.40 to 16.63. This is relevant for a multi-sector model for which the heterogeneity can be directly reflected in the calibration. For a one sector model, to limit the heterogeneity bias, a proper aggregation of disaggregated heterogeneous elasticities is desirable. Doing so results in an aggregate elasticity that takes values between 2 and 2.48.

The rest of this working paper is structured as follows. In the next section, we present the Feenstra (1994)'s methodology. Starting from the theoretical foundation, it moves to the empirical specification. Then, Section 3 presents the data sources and provides some descriptive statistics. Section 4 is dedicated to the baseline results, their sensitivity, and compare them to those of the existing literature. Section 5 estimates aggregate elasticities and Section 6 concludes.

3 BACI (Base pour l'Analyse du Commerce International) is a database provided by CEPII (Centre d'études prospectives et d'informations internationales). It can be accessed at: http://www.cepii.fr/CEPII/fr/bdd_modele/bdd_modele_item.asp?id=37

2. Methodology

This section describes the procedure developed by Feenstra (1994) to estimate the Armington elasticity. After discussing its theoretical foundation, we turn to the empirical implementation and the method used to estimate the elasticity.

2.1. Theoretical Underpinning

To start with, we assume that there is a representative household in Belgium that has preferences over different sectors[4] k *(k=1, 2, …, K)* through a Constant Elasticity of Substitution (CES) aggregator. We define the aggregate consumption C as a function of consumption categories C_k[5]:

$$C_t = \left[\sum_{k \in K} \alpha_{kt}^{1/\gamma} C_{kt}^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}} \tag{1}$$

In (1), $\alpha_{kt} > 0$ (with $\sum_{k \in K} \alpha_{kt} = 1\ \forall\ t$) is an exogenous preference parameter and it is allowed to vary across sectors and time; $\gamma > 1$ is the elasticity of substitution between sectors in Belgium.

We further assume that the sector-specific consumption basket (C_k) is in turn a CES aggregator over domestic and imported varieties of products. Here, we make use of the Armington assumption which conjectures that each sector product k is differentiated across its origins. Hence, for each sector k, the origin of the product, denoted by i, constitutes its variety. For domestic products, *i=Belgium.* We thus write C_k as a function of the demand for sector product k produced in country i, C_{ki}:

$$C_{kt} = \left[\sum_{i \in I_k} \beta_{kit}^{\frac{1}{\sigma_k}} C_{kit}^{\frac{\sigma_k - 1}{\sigma_k}}\right]^{\frac{\sigma_k}{\sigma_k - 1}} \tag{2}$$

In (2), $\beta_{kit} > 0$ is an exogenous preference parameter that denotes a taste or a quality shock. It is allowed to vary across products, their origins i, and through time. We do not assume that $\sum_{i \in I_k} \beta_{kit} = 1\ \forall\ t, \forall\ k$ for estimation convenience as discussed below. I_k is the set of countries of origin which supply product k, including the home country. $\sigma_k > 1$ is the elasticity of substitution between domestic and foreign varieties of a product. This is our parameter of interest. It is allowed to vary across products.

The representative household in Belgium chooses consumption of product k by maximizing utility given by (2) subject to the constraint that he/she spends all his/her budget on buying the domestic and imported varieties of sector products. His/her budget constraint reads:

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[4] In this working paper, a sector is defined in two ways depending on the level of (dis-)aggregation considered for the estimation. In Section 4 for instance, the estimation is performed at the HS 6-digit level. Therefore, the sector is a 6-digit HS product. By contrast, products are aggregated by group of products in accordance with the CPA nomenclature in Section 5.1. In this case, a sector is defined as a CPA group of products.

[5] In a multi-country setting where the estimations are performed for different countries, $j \epsilon J$ would index an importing country. In this case, consumption of country j would be C_j while the consumption of a sector-specific product in country j would be C_{kj}. Likewise, all parameters are indexed by j. But in this application, since we focus solely on Belgium, j is fixed and indexes Belgium. We therefore omit j to ease the notation. It is introduced later in Subsection 5.2 for the sake of clarity.

$\sum_{i \in I_k} P_{kit} C_{kit} = P_{kt} C_{kt}$

where P_{kit} is the price of product k originating from country i in period t and P_{kt} is the aggregate price of product k in period t.

The optimal consumption basket of the household is given by:

$$C_{kit} = \beta_{kit}^{\sigma_k - 1} P_{kit}^{-\sigma_k} P_{kt}^{\sigma_k} C_{kt} \tag{3}$$

where $P_{kt} = \left[\sum_{l \in I_k} \beta_{kit} P_{klt}^{1-\sigma_k}\right]^{\frac{1}{1-\sigma_k}}$ is the Dixit-Stiglitz price index.

It is further assumed that all varieties incur an iceberg transport cost τ_{ki} such that $\tau_{ki} > 1$ for imported products ($i \neq Belgium$) and $\tau_{ki} = 1$ for domestically produced products ($i = Belgium$). Following Samuelson (1954), this has become a standard assumption in international trade models. It allows to incorporate transport costs without explicitly modelling the transport sector.

With an iceberg transport cost, $P_{kit} = \tau_{kit} P_{kit}^{fob}$, where P_{kit}^{fob} represents the Free On Board (FOB) price of variety $i \in I_k$[6], which is also the price measure used in the BACI dataset. Hence, the introduction of an iceberg transport cost helps explaining the difference between the price of a product across destinations.

In practice, we do not observe the true prices. Instead, unit values, defined as import value divided by import quantity, are used as a proxy for prices. Unfortunately, unit values suffer from compositional effects since products with different characteristics are sometimes put together. Therefore, using unit values in the place of prices may introduce a measurement error in prices. To attenuate part of the measurement errors, the import demand given in Equation (3) is usually written in terms of market shares instead of quantities or values (Feenstra (1994), Broda and Weinstein (2006)). This way, the unit value will appear in both the numerator and the denominator so that the impact of the measurement error is reduced.

Let $s_{kit} \equiv \frac{P_{kit} C_{kit}}{P_{kt} C_{kt}}$ denote the share of expenditures on product k imported from country i. The import demand equation becomes:

$$s_{kit} = \beta_{kit}^{\sigma_k - 1} P_{kit}^{1-\sigma_k} P_{kt}^{\sigma_k - 1} \tag{4}$$

Applying the logarithmic transformation on Equation (4) and then differencing it in time results in the following equation:

$$\Delta \ln(s_{kit}) = (1 - \sigma_k) \Delta \ln(P_{kit}) + \Phi_{kt} + \varepsilon_{kit} \tag{5}$$

Notice that $\Phi_{kt} = (\sigma_k - 1) \Delta \ln(P_{kt})$ is independent of i. Hence, it is a time varying intercept that is common across all varieties of the product. It captures all the factors that affect the market share but are common to all supplying countries. $\varepsilon_{kit} = (\sigma_k - 1) \Delta \ln(\beta_{kit})$ is an error term. Because it depends on the

[6] Here, it is implicitly assumed that trade policies are fully passed onto the prices at the border. As such, the method developed by Feenstra (1994) does not allow an explicit assessment of the effects of trade policies (Fontagné et al. (2022)).

preference parameter β_{kit}, it captures changes in preferences unrelated to price changes. If $\sum_{i \in I_k} \beta_{kit} = 1 \; \forall \, t, \forall \, k$, the errors would be correlated across $i \in I_k$. In that case, a consistent estimation would require an estimator such as the Generalized Least Squares (GLS). This would require assumptions about the structure of the correlated error terms, making the estimation procedure complex. It would also require a balanced panel data at the product level. For these reasons, we do not impose that restriction here[7].

Thanks to the CES aggregator, we can match the elasticity of substitution between varieties of products in Equation (2) and the price elasticity of imports as in Equation (5). More precisely, if σ_k is the Armington elasticity and η_k is the trade elasticity, then $\eta_k = 1 - \sigma_k$[8]. To interpret Equation (5) as an import demand equation however, σ_k must be (strictly) higher than 1. Otherwise, the import demand equation would be upward sloping, which is inconsistent with economic theory[9]. Moreover, estimating σ_k requires tackling above all the issue of endogeneity. This is addressed in the next subsection.

2.2. Empirical Strategy

Prices are endogenous in Equation (5) because they are simultaneously determined with market shares. The literature typically deals with this issue in two ways. First, gravity-type models estimate a version of Equation (5) where bilateral import flows are regressed on trade costs variables including geographical, cultural, historical, and economic distances among trading partners. Price variations in trade costs are used to identify the Armington elasticity in these models. Identification is achieved by applying the instrumental variable approach (e.g., Fontagné et al., 2022) or the triple-difference method (Caliendo and Parro, 2015). Following Feenstra (1994), the second approach opt for a structural equation modelling by building a system of equations. It also exploits the panel structure of trade data to construct internal instruments. This way, the framework does not require the use of external instruments which are difficult to find in practice. In this subsection, we describe the empirical specification and the estimation method that will allow us to estimate the Armington elasticity using the framework developed by Feenstra (1994).

2.2.1. Empirical Specification

To construct the system of equations, Feenstra (1994) added an import supply equation to the import demand one. He assumed a simple supply curve, which, written in log-difference terms, is given by:

$$\Delta \ln(P_{kit}) = \frac{\omega_k}{1 + \omega_k} \Delta \ln(s_{kit}) + \Omega_{kt} + \xi_{kit} \tag{6}$$

$\Omega_{kt} = \omega_k \Delta \ln(P_{kt} C_{kt}) / (1 + \omega_k)$ summarizes factors that are specific to the importing country for a given product k (importer's time fixed effects) as it is independent of the exporting country; and ξ_{kit} is a random error. The latter captures e.g. any random changes in a technology. $\omega_k \geq 0$ is the inverse supply

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[7] One can think about β_{kit} as the sum of two elements: $\beta_{kit} = \beta_{ki} + \epsilon_{kit}$. β_{ki} indicates share parameters and sums to one. It captures differences in the average level of tastes across varieties. and ϵ_{kit} is an idiosyncratic shock. The share parameters are constant over time so that they drop out in the estimation procedure after time differentiation. Hence, the error term in Equation (5) consists exclusively of ϵ_{kit}.

[8] We use this mapping throughout this working paper.

[9] An upward sloping demand can nonetheless happen for specific goods such as Giffen goods or Veblen goods though this is unlikely to occur at given the level of aggregation.

elasticity and it is assumed to be constant across all exporting countries. Though restrictive, this assumption is needed for identification within this framework, given the structure of the data. Relaxing this assumption will require to adapt the methodology and to exploit information on both import and export markets for the same product. This is beyond the scope of the working paper. The reader interested in heterogenous supply elasticities is referred to Soderbery (2017) for further details.

Crucially, Feenstra (1994)'s methodology assumes that ξ_{kit} is independent of ε_{kit}. This means that once time specific factors of each product are controlled for in the import supply and import demand equations, the remaining demand and supply factors are assumed to be uncorrelated. Together, equations (5) and (6) form the system of equations that is used to determine prices and quantities simultaneously. Their estimation allows to assess the value of the Armington elasticity of substitution between varieties.

The framework exploits the fact that both Φ_{kt} and Ω_{kt} do not depend on supplying country factors. Thus, taken at the product level, they are only time varying. Hence, by differencing equations (5) and (6) with respect to a reference country l that is exporting to Belgium, it is possible to eliminate them from the system[10]. Doing so leads to equations (7) and (8) below:

$$\Delta^l \ln(s_{kit}) = (1-\sigma_k)\Delta^l \ln(P_{kit}) + \varepsilon^l_{kit} \tag{7}$$

$$\Delta^l \ln(P_{kit}) = \frac{\omega_k}{1+\omega_k}\Delta^l \ln(s_{kit}) + \xi^l_{kit} \tag{8}$$

where $\Delta^l x_{kit} = x_{kit} - x_{klt}$, $\varepsilon^l_{kit} = \varepsilon_{kit} - \varepsilon_{klt}$ and $\xi^l_{kit} = \xi_{kit} - \xi_{kit}$.

Under the assumption that ε^l_{kijt} and ξ^l_{kijt} are independent and hence uncorrelated, Equation (7) can be multiplied by Equation (8) to get:

$$Y_{kit} = \theta_{1k} X_{1kit} + \theta_{2k} X_{2kit} + u_{kit} \tag{9}$$

where:

$$\theta_{1k} = \frac{\omega_k}{(1+\omega_k)(\sigma_k - 1)} \qquad \text{and} \qquad \theta_{2kj} = \frac{1-\ \omega_k(\sigma_k - 2)}{(1+\omega_k)(1-\sigma_k)}$$

- $Y_{kit} = (\ \Delta^l \ln(P_{kit}))^2$
- $X_{1kit} = (\Delta^l \ln(s_{kit}))^2$
- $X_{2kit} = (\Delta^l \ln(P_{kit})) * (\Delta^l \ln(s_{kit}))$
- $u_{kit} = \varepsilon^l_{kit}\xi^l_{kit}/(\sigma_k - 1)$

[10] The choice of the reference country is crucial for the methodology. Section 4.2 discusses its impact on the results.

To solve for structural parameters from reduced form coefficients, Feenstra (1994) found it easier to define a new parameter $\rho_k = \frac{\omega_k(\sigma_k-1)}{1+\omega_k\sigma_k}$ such that $0 \leq \rho_k < \frac{(\sigma_k-1)}{\sigma_k} < 1$. Written in terms of ρ_k, θ_{1k} and θ_{2k} become:

$$\theta_{1k} = \frac{\rho_k}{(1-\rho_k)(\sigma_k-1)^2} \qquad \text{and} \qquad \theta_{2kj} = \frac{(2\rho_k-1)}{(1-\rho_k)(\sigma_k-1)}$$

ρ_k is interpreted as the correlation between the vertical shift in the demand curve due to the ε_{kit}^{l} shocks and the change in the equilibrium price. Its value is equal to zero when ω_k is also equal to zero. Further details on the link between structural estimates (σ_k and ω_k) and reduced form estimates (θ_{1k} and θ_{2k}) are provided in Appendix C.

With the aim of estimating Equation (9) using data that are typically obtained from trade databases, the next subsection discusses how to obtain not only consistent estimates of θ_{1k} and θ_{2k} but also economically feasible estimates of σ_k and ω_k.

2.2.2. Estimation Method

The estimation of reduced form coefficients in Equation (9) faces two endogeneity issues: simultaneity bias and measurement errors. The goal of this subsection is to discuss the estimation strategies that would allow to overcome them.

First, from equations (7) and (8) the prices and expenditure shares are correlated with the errors ε_{kit} and ξ_{kit} because of the simultaneity bias[11]. As a result, the error term u_{kit} is in turn correlated with X_{1kit} and X_{2kit} in Equation (9). To overcome the resulting endogeneity bias, Feenstra (1994) took advantage of the panel nature of the data. He pointed out that one can average Equation (9) to obtain a consistent estimator. This is equivalent to a Two Stage Least Square (TSLS) procedure where country dummies are used to predict the averages of X_{1kit} and X_{2kit} in the first stage. Here, the identification assumption requires that the relative variances of the demand and supply shocks vary across countries, in which case the regressors are not collinear. Furthermore, the inverse of the estimated residuals can be used as weights to control for heteroskedasticity.

Second, the use of import shares instead of import quantities or values is not sufficient to completely control for measurement errors. This is because unit values are themselves measured with errors. However, as shown by Feenstra (1994), if the variances of the measurement error are assumed to be constant across supplying countries, a constant term added to Equation (9) would further control for measurement error. This is because Y_{kit} is the second moment of the unit values and in large samples it will equal the variance of the true price plus the variance of the measurement error. Thus, the included constant term will equal the variance of the measurement error.

So, Equation (9) is averaged over time so that the variables are constant over time but not across countries, and a constant term is included in the regression:

[11] This can be seen by solving the reduced form price and market shares.

$$\bar{Y}_{ki} = \theta_{0k} + \theta_{1k}\bar{X}_{1ki} + \theta_{2k}\bar{X}_{2ki} + \bar{u}_{ki} \quad (10)$$

Where $\bar{Y}_{ki} = \frac{1}{T_k}\sum_{t=1}^{T_k} Y_{kit}$, $\bar{X}_{1ki} = \frac{1}{T_k}\sum_{t=1}^{T_k} X_{1kit}$, $\bar{X}_{2ki} = \frac{1}{T_k}\sum_{t=1}^{T_k} X_{2kit}$, $\bar{u}_{ki} = \frac{1}{T_k}\sum_{t=1}^{T_k} u_{kit}$, θ_{0k} is the constant term and T_k is the number of period for product k.

Once estimates of the reduced form coefficients θ_{1k} and θ_{2k} are obtained, estimates of σ_k and ω_k could be recovered. Hence, the methodology proposed by Feenstra (1994) allows to tackle simultaneity and measurement error biases without the need to resort to external instruments. Because of the difficulty in finding valid instruments in practice and because trade datasets have a panel structure, it has gained in popularity. Besides, the estimation of Equation (10) will allow us to assess the presence of measurement errors at the product level since any statistically significant constant term would be an indication of that issue.

Achieving consistent estimates of θ_{1k} and θ_{2k} does not guarantee economically meaningful results, however. Actually, it may happen that $\sigma_k < 1$ and/or $\omega_k < 0$, leading to an upward sloping demand curve and/or a downward sloping supply curve. Feenstra (1994) did not discuss explicitly how to handle them except stressing that they will happen when θ_{1k} is negative. In order words, his framework provides economically meaningful estimates of σ_k and ω_k only when θ_{1k} is positive. Broda and Weinstein (2006) refined the estimation procedure by adding a grid search. More specifically, whenever their estimates are economically infeasible, they search for parameter values by evaluating their objective function, a Generalized Method of Moments (GMM) objective function, only within a plausible parameter space ($\sigma_k > 1$ and $\omega_k > 0$).

Later on, Soderbery (2015) demonstrates that outliers are overweighted in the procedure of Feenstra (1994) while the grid search estimation of Broda and Weinstein (2006) suffers from constrained search inefficiencies: small sample biases, convergence issues, and a perverse polarization of the elasticity in actual data. He then advocates for the Limited Information Maximum Likelihood (LIML) method, which seems to correct for these issues. The resulting LIML estimator follows Feenstra (1994) to control for measurement error and heteroskedasticity but applies a constrained non-linear LIML when infeasible estimates occur.

In this paper, we incorporate a constrained optimization into the TSLS procedure of Feenstra (1994). We also use the hybrid LIML estimator proposed by Soderbery (2015). But as the results will show, the TSLS method will be preferred. This is because, in contrast to the (hybrid) LIML estimator, it leads to estimates that are more consistent with all the model constraints.

To sum up, we follow Feenstra (1994) by using the two-step GMM procedure to estimate reduced-form coefficients. If the estimate returns a positive value of θ_{1k}, we recover the structural parameters. In case θ_{1k} is negative, we incorporate the constraints that $\sigma_k > 1$ and $0 < \rho_k < 1$ into the optimization process to get estimates of ω_k and σ_k. All estimations are performed at the product level. Before discussing the results, we first turn to the BACI dataset, the dataset that allows us to perform such estimations.

3. Data

To estimate Armington elasticities, we use the BACI database which is constructed by the CEPII. In this section, we first explain the characteristics of the database before turning to the descriptive analysis for highly disaggregated data as well as for relatively aggregated data.

3.1. Sources and Sample

On the basis of data from the COMTRADE database, the BACI database provides yearly information on bilateral trade flows on the 6-digit level of the harmonized system (HS6). This consists of highly disaggregated data covering more than 5,000 products. The database is freely available, and it covers more than 200 countries over the period 1996 – 2021. All trade flows in BACI are reported Free On Board (FOB, net of transport, insurance, and freight costs) in thousands of current US dollars. The database has been used by several studies including Imbs and Méjean (2010 and 2017), Fontagné et al. (2022), and Kastrup et al. (2021).

Compared to COMTRADE, the main advantage of BACI is that it reconciles trade flows to facilitate international comparison[12]. This is done in two ways. First, BACI makes use of mirror flows to fill missing data. It does so by exploiting differences between the information reported by the importer and the exporter of the same product. But since imports are usually reported CIF (cost insurance freight) while exports are reported FOB (free on board), CIF rates are removed from import flows, which requires an estimation of CIF rates. Under the assumption that they are strongly correlated with direct measures of shipping costs, CIF/FOB ratios are estimated using a gravity-type equation. That is, they are estimated as a function of variables such as distance, contiguity, or landlockedness. The resulting estimated CIF rates are removed from import flows, a technique known as *fobization*. As a result, import and export flows as well as unit values obtained with the BACI are coherent and thus easily comparable across countries.

Second, to ensure harmonization in quantities, BACI converts all quantities into tons using mirror flows again. That is, when a given product is reported in tons by one country and in different units by another, the rates of conversion of the different units into tons are estimated using mirror flows. These implicit rates of conversion on heterogenous units are used to convert them into tons[13]. This way, the quantities provided by BACI are comparable across countries as well.

Several versions of the BACI database are available: HS0 (from 1988 to 2021), HS1 (from 1996 to 2021), HS2 (from 2002 to 2021), HS3 (from 2007 to 2021), HS4 (from 2012 to 2021), and HS5 (from 2017 to 2021)[14]. Among them, we choose the HS3 version despite the availability of longer series in older versions. This is because in contrast to the latter, the HS3 version has a direct comparison with CPA 2008 facilitating a

[12] See Gaulier and Zignago (2010) for a detailed documentation of the BACI database.

[13] Concerning quantities reported in unknown units or in Kwh, they are dropped from the database for simplicity because goods that are typically recorded in kwh such as electricity or gas may not be well covered by custom controls. In addition, these goods are difficult to store. Moreover, the conversion is only performed if a minimum of 10 mirror flows have been used in its computation, and if the standard deviation of the conversion rates is inferior to 2.5 About 8.5% of final quantities in BACI have been converted using this method.

[14] The period of coverage of the corresponding version of the dataset is in parentheses.

comparison between our estimates at the level of HS products and those at the level of CPA products. Nonetheless, we will test the robustness of our results to using an older version of the BACI database[15].

We extract only those data from BACI where Belgium is the importer. In addition, we do not make use of the data after 2019 for the estimation to avoid any potential effects of the COVID-19 pandemic on the results. Therefore, our data cover the period 2007 – 2019.

3.2. Descriptive Analysis

A number of comments emerge from the BACI dataset. First, Belgium has imported many products from many partners. Each year over the period 2007 to 2021, it has imported almost 5,000 products (see Table 2). As shown on Graph 1, this amounts to an annual value of total imports that has fluctuated around $ 300 billion over the period. With a value of about $ 400 billion in 2007, Belgian imports dropped in 2008 following the Global Financial crisis. Since then, they have been characterized by small increases or decreases until the COVID-19 pandemic, though the impact of the health crisis is limited compared to the financial crisis.

It can also be seen in Table 1 that, over the same period, more than 200 partners have exported to Belgium on the annual basis. These include certain insignificant partners such as Guatemala or South Soudan whose share of products in Belgian imports is less than 0.0001%. At the other extreme, Netherlands is the most important import partner, followed by Germany, France, the UK and Italy (Graph 1). Unsurprisingly, all of them are members of the European Union, though the UK has recently left the regional economic integration area. Also, the main three partners consist of neighbouring countries. Together, they account for 41 to 48% of Belgian imports[16].

Table 1 Number of products imported and number of partners from 2007 to 2021

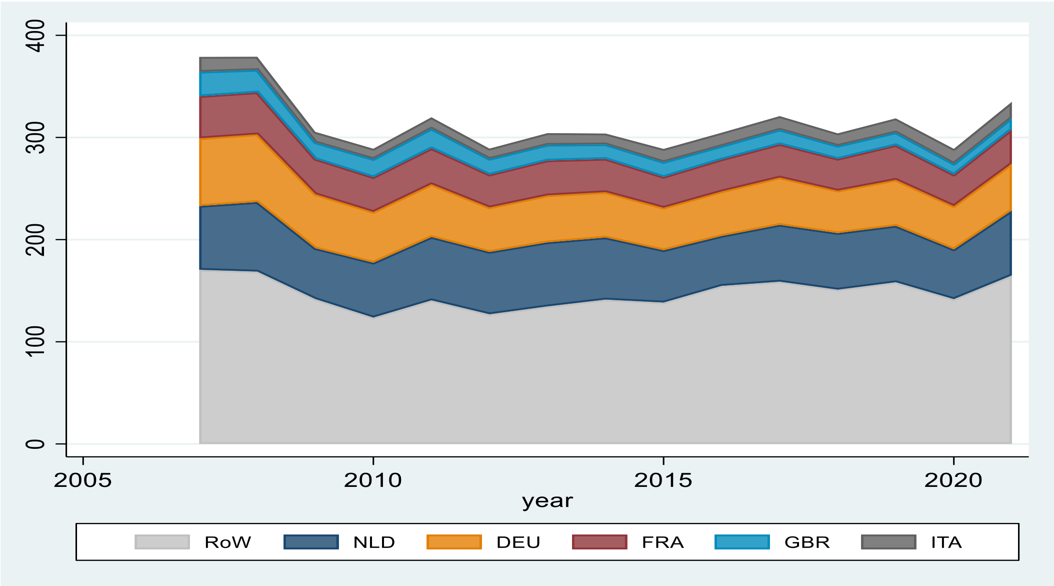
Year	Number of products	Number of partners	Year	Number of products	Number of partners
2007	4962	209	2014	4913	212
2008	4961	216	2015	4916	211
2009	4963	210	2016	4905	216
2010	4955	211	2017	4909	217
2011	4960	211	2018	4905	221
2012	4931	214	2019	4904	221
2013	4916	217	2020	4898	216
			2021	4891	223

Source: Own calculations based on BACI. This table shows the number of HS 6-digit products imported to Belgium and the number of partners exporting to Belgium from 2007 to 2021.

[15] The correspondence table between HS3 and CPA2008 codes is extracted from RAMON of EUROSTAT.

[16] In principle, the BACI database records trade flows on products, including goods and services. Its record of services is however very limited. This reflects the difficulty in recording services flows in international trade datasets.

Graph 1 Imports of Belgium from main partners (value in billions of euros)



Source: Own computations with data from BACI.
Note: Import flows are deflated using the elementary unit value index. Rest of the world (RoW) is defined as the world except Netherlands (NLD), Germany (DEU), France (FRA), United Kingdom (GBR), and Italy (ITA).

Second, at the 6-digit HS product level, the average number of partners is high as well. As reported in Table 2, the average number of partners at the product level is 70 while the median is 65. Both the mean and the median of the number of years for which Belgium imported products are about 13. For consistency of the results, we have retained only products with more than 5 years of observations and with more than 10 trading partners.

Table 2 also reports descriptive statistics at the product level for import share and unit price. The main message that emerges from that table is that observations at the product level display a large dispersion. This holds for each of the variables in the table. For instance, despite an average value of 0.040%, the import share ranges from a value very close to 0 to more than 9%. The dispersion is especially high for unit prices which display a standard deviation of more than 1 billion, reflecting the heterogeneity of the products covered in the database. The maximum value of the unit price is over 1 trillion US $ and is observed for "diamonds"[17].

Table 2 Descriptive Statistics at the product level

	Mean	Median	Standard Dev.	Minimum	Maximum	Unit
Number of partners	70.170	65	29.185	10	215	-
Number of periods	12.986	13	0.289	5	13	-
Import share	0.040	0.002	0.190	0	1	%
Unit prices	5407.24	13.646	1,276484	0	1.011e+09	1,000 US $

Note: This table shows descriptive statistics of the sample. They are computed over the period 2007 - 2019.

Next, we turn to the description of CPA product groups (at the level of A64). For each CPA product group, Table 3 reports the number of HS products included in the aggregation, the import share of the product category and the unit value computed at that level of aggregation. Two remarks are in order.

[17] The corresponding HS code is 710239 which stands for *"Diamonds, whether or not worked, but not mounted or set.- Others"*

First, out of the 64 CPA product groups, only 30 are matched with at least one HS 6-digit products. The missing groups tend to be related to services. Moreover, after aggregation, goods-related categories tend to be more represented than services-related ones. For instance, with respectively 810 and 720 HS products, *Textiles, wearing apparel, leather and related products (C13-15)* and *Chemicals and chemical products (C20)* are the product categories with the highest HS number of products. At the same time, only 1 or 2 HS 6-digit products are matched in *Printing and recording services (C18)*, *Architectural and engineering services; technical testing and analysis services (M71)*, *Other personal services (S96)*, and *Other professional, scientific and technical services and veterinary services (M74-75)*. This reflects the dominance of tradable merchandises in trade databases.

Second, dispersion is present here as well. For example, the unit value at the CPA product categories ranges from $ 227 to about $ 200,000. Though large, this range is narrower than the one obtained across HS 6-digit products. This is probably the result of aggregation which is masking composition of trade flows (Imbs and Mejean, 2015).

Taken together, the dispersion in Table 2 and in Table 3 suggests that products contained in the BACI dataset have different characteristics. This highlights the importance of performing the estimation at the product level to avoid aggregation bias. In the next section, we will use the data from BACI to estimate the Armington elasticity across the 6-digit HS products and across CPA product groups. Moreover, we will aggregate the latter to obtain a unique elasticity aggregated at the level of the economy.

Table 3 Descriptive Statistics across CPA product groups

CPA codes	Labels of CPA product groups	Number of HS goods	Import share	Unit value
A01	Products of agriculture, hunting and related services	220	3.08%	0.729
A02	Products of forestry, logging and related services	16	0.17%	0.227
A03	Fish and other fishing products; aquaculture products; support services to fishing	5	0.00%	1.599
B	Mining and quarrying	95	12.33%	0.572
C10-12	Food, beverages and tobacco products	487	7.36%	1.137
C13-15	Textiles, wearing apparel, leather and related products	810	4.05%	10.611
C16	Wood and of products of wood and cork, except furniture; articles of straw and plaiting materials	85	0.72%	0.586
C17	Paper and paper products	126	1.63%	1.080
C18	Printing and recording services	1	0.05%	8.201
C19	Coke and refined petroleum products	20	0.67%	0.477
C20	Chemicals and chemical products	780	14.36%	1.405
C21	Basic pharmaceutical products and pharmaceutical preparations	100	8.92%	79.849
C22	Rubber and plastic products	136	2.89%	4.486
C23	Other non-metallic mineral products	165	1.14%	0.525
C24	Basic metals	354	5.59%	1.434
C25	Fabricated metal products, except machinery and equipment	244	2.24%	4.209
C26	Computer, electronic and optical products	265	4.70%	64.024
C27	Electrical equipment	186	2.79%	10.771
C28	Machinery and equipment n.e.c.	467	6.96%	11.748
C29	Motor vehicles, trailers and semi-trailers	64	10.73%	11.094
C30	Other transport equipment	84	1.43%	5.974
C31-32	Furniture and other manufactured goods	193	5.28%	15.830
D	Electricity, gas, steam and air conditioning	2	0.29%	199.285
E37-39	Sewerage services; sewage sludge; waste collection, treatment and disposal services; materials recovery services; remediation services and other waste management services	74	2.04%	0.800
J58	Publishing services	265	4.70%	4.980
J58	Publishing services	21	0.29%	59.222
J59-60	Motion picture, video and television programme production services, sound recording and music publishing; programming and broadcasting services	8	0.20%	78.593
M71	Architectural and engineering services; technical testing and analysis services	1	0.00%	26.911
M74-75	Other professional, scientific and technical services and veterinary services	2	0.00%	89.026
R90-92	Creative, arts, entertainment, library, archive, museum, other cultural services; gambling and betting services	7	0.06%	0.716
S96	Other personal services	1	0.00%	0.729

Note: This table reports the number of the 6-digit HS products matched in each CPA product groups. It also reports the import share of each CPA product group as well as the corresponding unit value. The latter are computed as import divided by import quantity at the product group level.

4. Highly Disaggregated Elasticities

In this section, we present the results of highly disaggregated Armington elasticities. We start with the baseline results which are supplemented with some robustness analyses. We then compare our results to those of the existing literature.

4.1. Baseline Results

We estimate the Armington elasticity using three different methods: OLS ignoring endogeneity, TSLS as suggested by Feenstra (1994), and the LIML suggested by Soderbery (2015). We choose the reference country as the partner with the highest share in Belgium's imports among those with the longest trade transactions over the period of study. All estimations in this subsection are performed at the level HS 6-digit and the descriptive statistics of the estimates are summarized.

To illustrate the threat caused by endogeneity issues, we start with the OLS estimates. Those are obtained by estimating Equation (5) alone. Except for controlling for country fixed effects, these estimates ignore endogeneity issues. The results are summarized in Table 4. On the first row of the table, we report all estimates of the elasticity, no matter their value. Both the mean and the median are close to 0.7. Hence, according to these position measures, the Armington elasticity obtained with OLS is economically infeasible since it implies a positive slope of the import demand function. Only 17% of the estimates obtained with OLS are economically feasible. Moreover, the OLS estimator does not seem to really differentiate estimates of Armington elasticity across products as indicated by the low values of the standard deviation of the estimates and their narrow range in Table 4.

On the second row of Table 4, we report the summary statistics for economically feasible estimates only. That is, estimates with a value of $\sigma_k < 1$ are dropped from the summary statistics. As a consequence, both the mean and the median increase, but they remain close to 1. Altogether, the results in that table suggest that the OLS estimates are unreliable.

The unreliability of the OLS estimates is not surprising since it faces at least two endogeneity issues: a measurement error since unit values are used in the place of prices, and a simultaneity bias coming from the correlation between import demand shock and prices as well as market shares. With OLS, only the import demand equation is estimated without any control to the endogenous prices. Overall, the results in Table 4 suggest that these sources of endogeneity are so perverse that they make OLS estimates economically meaningless.

Table 4 Summary of the Armington elasticities estimated with OLS, ignoring endogeneity

	N	Mean	Median	Standard Dev	Minimum	Maximum
All σ_k	4,397	0.775	0.76	0.187	0.406	1.248
Only $\sigma_k > 1$	746	1.2	1.134	0.204	1.001	1.248

Note: The estimates are obtained with the OLS estimator performed at the product level. The results are trimmed by 10: the lowest 5 and the highest 95 values of are σ removed. The reference country is the dominant supplier.

We therefore move to the Feenstra's methodology which tackles endogeneity properly. As explained previously, it solves the simultaneity bias by supplementing the import demand equation with an import supply equation. Both the Armington elasticity (σ_k) and the inverse supply elasticity (ω_k) are estimated as part of the resulting system of equations. Moreover, thanks to the panel dimension of the dataset, the methodology uses country-specific factors as instruments and estimates reduced-form coefficients with TSLS before recovering structural parameters. The framework also allows to attenuate the measurement error bias. Finally, it controls for heterogeneity by using the inverse of the estimated residuals as weights.

The results are reported in Table 5 with the reduced-form coefficients in Panel A and the structural parameters in Panel B. Among the reduced-form coefficients, only the constant term (θ_{0k}) has a meaningful interpretation because its significance indicates the presence of measurement errors. Graph 6, which shows the share of significant reduced-form coefficients for different levels of significance along with a distribution of their p-value, indicates that, at the 10% statistical significance level, more than 92% of the estimated θ_{0k} are statistically significant. In other words, the measurement error tends to represent a serious threat for almost all the products covered in the estimation process.

Looking at the structural parameters, the raw estimates indicate a mean of -4.36 and a median of 4.52 for the Armington elasticity. As indicated by the mean and the range, some estimates of σ_k are negative. Though θ_{1k} does not have a meaningful interpretation, it plays a crucial rule when applying the formula derived by Feenstra (1994) to recover the structural parameters. Because the values of σ_k and ρ_k derived by Feenstra make sense economically only when θ_{1k} is positive. For this reason, the second row of Panel B restricts the sample to observations whose θ_{1k} is statistically positive. This shifts the distribution of σ_k to the right. The average of σ_k becomes 6.46 and the median becomes 4.93. In addition, the values of σ_k are now all positive and the range has shrunk, with a minimum and a maximum of 1.12 and 70.64 respectively. Nonetheless, several observations still have a negative value of ω_k. For this reason, the third row of Panel B deletes those observations. The results are virtually unchanged. This is confirmed on Graph 8 which plots the histograms σ_k corresponding to the first three rows of Panel B in Table 5.

Table 5 Summary of the Armington elasticities estimated with TSLS

	N	Mean	Median	Standard Dev	Min	Max
Panel A: Reduced-form coefficients						
θ_{0k}	4,065	1.358	0.750	2.578	-8.39	74.158
θ_{1k}	4,065	.057	0.058	.241	-7	3.061
θ_{2k}	4,065	.36	0.375	.524	-3.493	7.073
Panel B: Structural Parameters						
All σ_k	3,699	-4.361	4.515	322.453	-17,538.5	2,560.87
σ_k with $\theta_{1k} > 0$	2,256	6.458	4.925	5.181	1.124	70.694
σ_k with $\theta_{1k} > 0$ and $\omega_k \geq 0$	2,152	6.336	4.860	4.967	1.124	70.694
ω_k	2,152	10.834	1.205	333.379	.027	15,426.561
ρ_k	2,152	.644	0.673	.191	.028	.969

Note: The estimates are obtained with the TSLS estimator performed at the HS 6-digit product level. The observations are weighted with the inverse of the variance of estimated residuals. Standard errors of the structural parameters are obtained with the Delta method.

The corresponding values of ω_k and ρ_k are reported on the last two rows of the table. The first ranges from 0.03 to more than 15,000. Its average is 10.83 and its median is 1.21. With an average of 0.64 and a median of 0.67, the second takes values from 0.03 to 0.97.

Soderbery (2013) argues that the TSLS estimator used above is biased in small samples. Because trade datasets are typically of small samples, he suggested to use the LIML estimator. The main difference with respect to the TSLS procedure is that the data are weighted with the variance of the true errors, which is a function of the reduced-form parameters θ_{1k} and θ_{2k}. In this sense, the LIML estimator is nonlinear (Feenstra et al. (2018)).

We also estimate the reduced-form coefficient with the LIML estimator as well and report the results in Table 6. Here, the measurement error is present for 70% of products. As before, the raw data produces implausible estimates of the structural parameters. When we focus on the observations with statistically positive θ_{1k}, all the estimates of σ_k are higher than 1. The resulting mean is 4.74 and the median is 3.05. When we further drop estimates with a negative value for ω_k, both the average and the median decrease to become 3.12 and 2.35 respectively.

Compared to the results with TSLS procedure, there are two main differences. First, the estimates of σ_k obtained with the LIML estimator are considerably lower. The mean and the median of the LIML estimator are about half of those with the TSLS. Soderbery (2015) found similar results and attributed the difference to the fact that the TSLS suffers from small sample bias while the LIML does not. Second, only 5% of the results obtained with the TSLS have a negative ω_k. This share amounts to 34% with the LIML estimator. Hence, while the TSLS may suffer from small sample bias, the LIML tends to produce estimates that violate more often the constraint on ω_k.

Table 6 Summary of the Armington elasticities estimated with LIML

	N	Mean	Median	Standard Dev	Minimum	Maximum
Panel A: Reduced-form coefficients						
θ_{0k}	4,783	1.359	0.486	4.175	-25.87	185.24
θ_{1k}	4,783	.132	0.118	.664	-15.308	4.466
θ_{2k}	4,783	.306	0.336	1.944	-17.223	11.586
Panel B: Structural Parameters						
All σ_k	4,316	-1.575	2.421	726.708	-39,537.6	19,691.429
σ_k with $\theta_{1k} > 0$	2,383	4.736	3.048	5.819	1.065	103.315
σ_k with $\theta_{1k} > 0$ and $\omega_k \geq 0$	1,577	3.121	2.352	2.866	1.065	44.32
ω_k	1,577	5.543	0.919	41.15	.006	1,347.64
ρ_k	1,577	.398	0.381	.249	.002	.971

Note: The estimates are obtained with the LIML estimator performed at the HS 6-digit product level. The observations are weighted with the inverse of the variance of estimated residuals. Standard errors of the structural parameters are obtained with the Delta method.

Our next goal is to handle infeasible estimates properly. Feenstra (1994) did not discuss explicitly how to deal with unfeasible estimates. Nonetheless, he made it clear that θ_{1k} should be positive to obtain estimates of both σ_k and ρ_k (or ω_k) that make sense economically. Implicitly, this suggests getting rid of those estimates as is done in Table 5 and in Table 6.

In the literature, there are attempts to handle estimates that are economically infeasible directly in the estimation process. They typically proceed in two steps. First, they estimate the reduced-form coefficients and recover the structural parameters. In case the latter are economically infeasible, which happens when θ_{1k} is negative, they resort to an optimization problem which constrains the value of σ_k and ρ_k directly in the estimation process. For instance, Broda and Weinstein (2006) optimize a GMM problem defined from Equation (10) and search over $\sigma_k > 1$ and $\rho_k > 0$. Soderbery (2013) argues that the grid search method suffers from a polarization problem in the sense that the results it produces are most of the time either perfectly elastic or perfectly inelastic. He also criticizes the grid search method on the basis that it can miss the actual constrained optimum. He then advocated for a constrained TSLS and a hybrid LIML estimator though he preferred the latter.

We follow this procedure to incorporate the constraints into both the TSLS and the LIML estimators. But the constrained estimation procedure faces two main difficulties in practice. First, because it is called for when θ_{1k} is negative, it tends to produce estimates of θ_{1k} that are forced to be positive but close to 0. This results in either a very large value of σ_k or a value of ρ_k close to 0. For this reason, the results presented for constrained estimations are trimmed at the 10% level.

The second difficulty faced by the constrained estimation is technical. For practical reasons, the optimization is performed over σ_k and ρ_k instead of σ_k and ω_k. For this to be consistent, the condition $0 \leq \rho_k < \frac{\sigma_k - 1}{\sigma_k} < 1$ must hold. Indeed, $\rho_k < \frac{\sigma_k - 1}{\sigma_k}$ is needed to guarantee that ω_k is positive. Yet, $\rho_k < \frac{\sigma_k - 1}{\sigma_k}$ cannot be imposed in the optimization process[18]. As a result, despite resorting to a constrained estimation, we could still have a negative value for ω_k. We will drop such cases when presenting the results.

The results of the constrained estimation are reported in Table 7. As a result of introducing the constraints, the estimated values of σ_k in that table are all higher than 1. With the TSLS estimator (Panel A), the estimated σ_k is 7.04 on average and its median is 4.97. About 5% of the estimates on this row have a negative ω_k and dropping them does not really affect the results. The median is similar to that obtained in Table 5 when we focused on positive ω_k, but the average is slightly higher, and the range has widened.

In Panel B of Table 7, we replicate the hybrid LIML procedure of Soderbery (2015). The average of σ_k is 4.78 while the median is 3.19. More than 35% of the estimates have a negative ω_k. When we drop those observations, we obtain an average of 3.35 and a median of 2.48. The LIML has produced less dispersed estimates than the TSLS.

Table 7 Constrained estimation with TSLS and LIML

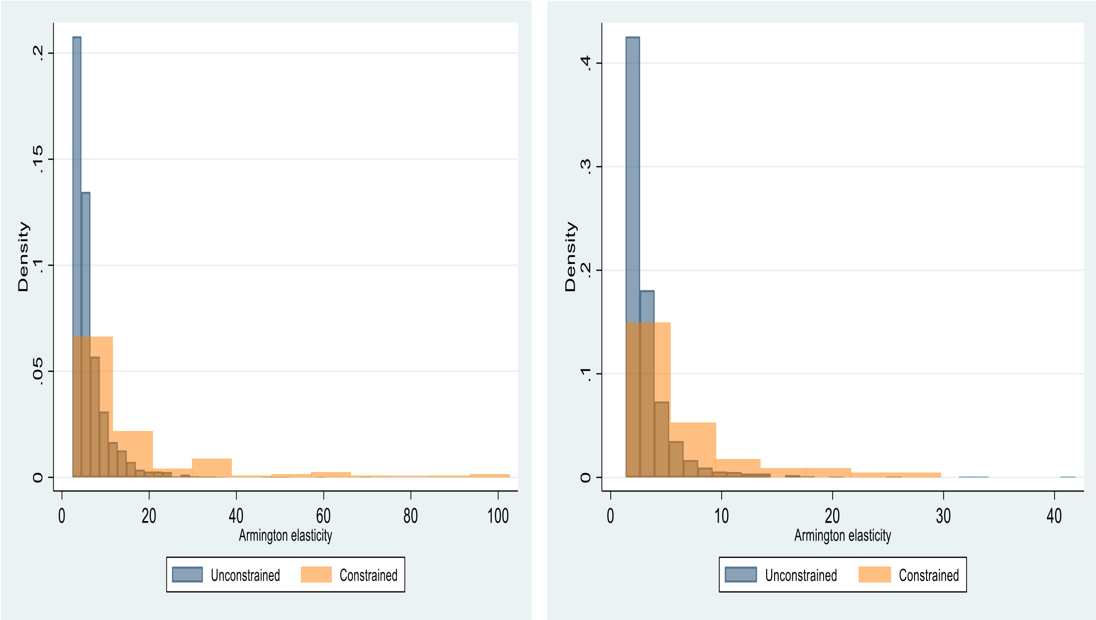
	N	Mean	Median	Standard Dev	Minimum	Maximum
Panel A: TSLS						
All σ_k	2,573	7.036	4.987	7.322	2.393	102.601
σ_k with $\omega_k \geq 0$	2,455	6.859	4.918	6.831	2.393	102.601
ω_k	2,455	9.675	1.115	312.141	.006	15,426.561
ρ_k	2,455	.645	0.670	.19	.01	.988
Panel B: LIML						
All σ_k	2,388	4.782	3.192	4.753	1.344	41.863
σ_k with $\omega_k \geq 0$	1,517	3.348	2.477	2.954	1.344	41.863
ω_k	1,517	5.766	0.953	41.943	.011	1,347.64
ρ_k	1,517	.425	0.412	.243	.007	.971

Note: The estimates are obtained with the LIML estimator performed at the HS 6-digit product level. The observations are weighted with the inverse of the variance of estimated residuals. Standard errors of the structural parameters are obtained with the Delta method.

[18] It is not clear whether this is due to the complexity of the model in hand and/or to the inability of the software (STATA) to handle such constraints. Future work will document this technical issue further.

For both the TSLS and the LIML estimators, the distribution of σ_k seems to be influenced by the incorporation of the constraints in the estimation procedure. This can be seen on Graph 2 which compares the distribution of σ_k when the constraint is used or not for both the TSLS (on the left panel) and the LIML (on the right panel) estimators. With both estimators, the use of the constrained optimization makes the distribution shift considerably to the right. This is particularly true for the TSLS.

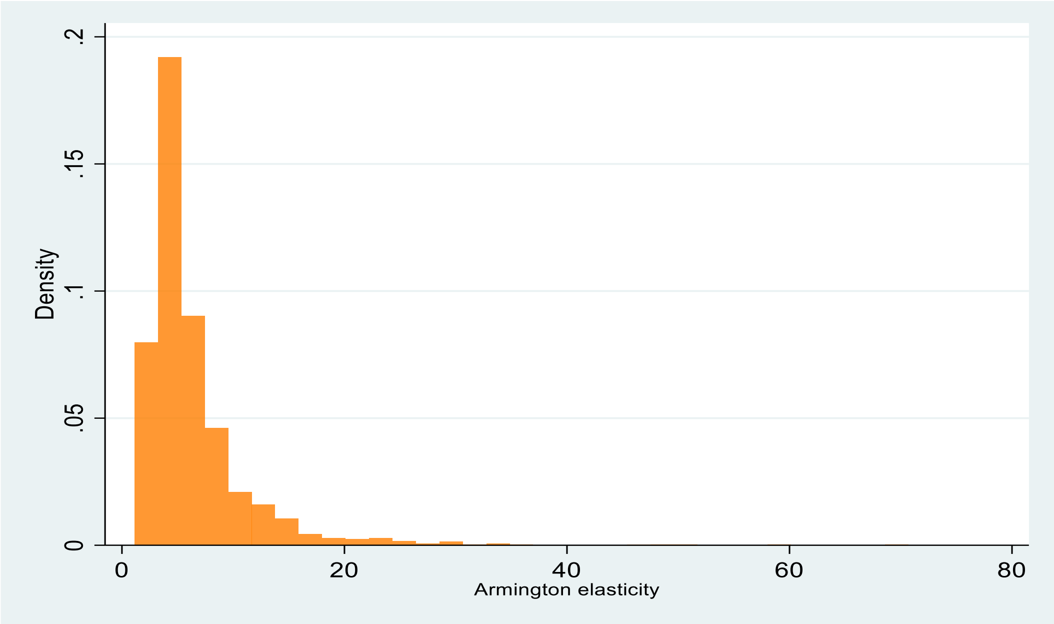
Graph 2 Comparison unconstrained and constrained estimations for TSLS (left) and LIML (right)



Note: This graph compares the Armington elasticities estimated at the HS6-goods level with and without the constraints. The estimates are obtained using the TSLS (left panel) and the LIML (right panel) estimators. The results are trimmed at the 10% level.

To sum up, though the TSLS estimator may suffer from small sample bias, it leads to estimates that are broadly more consistent with all constraints of the model than the LIML estimator. Moreover, in contrast to unconstrained estimates, constrained ones tend to produce more outliers. For these reasons, our preferred results are those obtained with the unconstrained TSLS. They were reported in Panel B of Table 5 and are plotted on Graph 3. The distribution of the adjusted r-squared associated with these results, plotted on Graph 9 in the appendix, seems satisfactory. So does the distribution of the p-values of associated to the Armington elasticity plotted on Graph 10. Indeed, at the 10%, almost 95% of the estimated elasticities are significantly higher than 1.

Graph 3 Distribution of Armington elasticities estimated at the HS 6-digit level - main results



Note: This graph plots the Armington elasticities estimated at the HS 6-digit product level using the unconstrained TSLS procedure. It corresponds to the elasticities obtained in the third row in Panel B of Table 5.

4.2. Robustness Checks

In this sub-section, we check the sensitivity of our results and run a couple of tests. We focus successively on the choice of the reference country, the version of the dataset, and the handling of unfeasible estimates.

A key aspect of the methodology advocated by Feenstra (1994) is the choice of the reference country. This is necessary to wipe out time fixed effects in equations (5) and (6). Feenstra (1994) advised to choose a trading partner that has supplied the home country every year, which helps keeping most observations in the estimation procedure. In addition, in the case of many potential candidates, he further recommends selecting the dominant supplier arguing that this choice will reduce measurement errors. In practice, this consists in choosing the trading partner with the highest import share. This choice is used for our baseline results.

Our first sensitivity test is to verify the extent to which our baseline results are affected by that choice. But the reference country should not be chosen randomly since the results of Mohler (2009) indicate that the most stable results are obtained when one chooses the reference country among the dominant suppliers. Thus, to perform our sensitivity test, we modify the rule by choosing the second dominant supplier instead of the first. The corresponding summary statistics are comparable to those of the baseline results. The average Armington elasticities across HS 6-digit products is 5.92 while the median is 4.69. The elasticity ranges from 1.45 to 66.27 with a standard deviation 4.38. Moreover, the distribution of the elasticities plotted on Graph 11 in the Appendix looks very similar to that of our main results (on Graph 3). This indicates that our baseline results are not really affected by moving from the first to the second dominant supplier.

Our next sensitivity test checks for the impact of the version of the BACI dataset. As discussed earlier, for the baseline results, we use the HS3 version of the dataset which covers the period 2007 – 2021 while the HS2 version with longer series (covering the period 2002-2021) was available. We now replicate our baseline results using the HS2 version of the dataset. The results are reported on Graph 12 in the Appendix. Descriptive statistics and the histogram of the obtained elasticities are comparable to those of the baseline results. In other words, our baseline results using the HS3 version of the BACI dataset are robust to using the HS2 version of the dataset.

Finally, while in the previous subsection we followed the literature and put constraints on σ_k and ρ to handle infeasible estimates, we now experience introducing a constraint directly on θ_{1k}. This way, focusing on θ_{1k} that are positive would allow us to apply Feenstra's formula directly. The results are reported on Graph 13 in the Appendix. Except from the fact that the range has widened, and the average has slightly increased, they are similar to the baseline results.

To sum up, our main results are robust to the choice of the reference country, to the use of an alternative version of the dataset, and to an alternative constrained estimation procedure.

4.3. Comparison to the Literature

We finish this section by comparing our results to those obtained in the literature. Such a comparison should be made with caution since any observed differences may come from several factors: databases, estimation methods, disaggregation levels, and geographic and possibly time coverages. In addition, the median is the statistic usually reported by studies using highly disaggregated data, but sometimes only the mean is reported. Hence, the median is our preferred statistic for the comparison, but we resort to the mean when it is not available.

With this in mind, Table 8 reports studies that use highly disaggregated data (more than 4 digits). The first column highlights the estimation methodology and shows that two methodologies are typically used in the literature: the system of equations modelling and the gravity-type models. The second and third columns of the table specifies the database and the disaggregation level respectively. The last two columns indicate the country covered by the study and the value of the elasticity.

The median Armington elasticities reported in Table 8 ranges from 1.9 to 4.9. Broda and Weinstein (2006) estimate the Armington elasticity using Feenstra's methodology on 10-digit goods imported into the US. They find a median value of 3.7 and 3.1 respectively for the periods 1972 – 1988 and 1990 – 2001. When aggregated at the 5-digit level, their medians become 2.8 and 2.7. Soderbery (2015) applies the same methodology to US data obtained from BACI and reports a median estimate of about 2 for the period 1993 - 2007. For European countries, Mohler and Seitz (2009) use disaggregated data from Eurostat and find an elasticity of 4.1 for Belgium and Luxembourg while the median value found in Corbo and Osbat (2013) is 3.2.

On the other hand, Hertel et al. (2007) and Fontagné et al. (2022) estimate the Armington elasticities with a gravity-type model. Hertel et al. (2007) estimates the elasticities for 7 non-European countries and found an average value that ranges from 1.8 to 10.1. Fontagné et al. (2022) uses the same dataset as us and reports a median value of 5. This is very close to our main result which shows a median elasticity of 4.9. Except from that, the median found in this paper tends higher than those found in the literature.

Table 8 Comparison to the literature – highly disaggregated elasticities

	Estimation methodology	Database	(Dis) Aggregation level	Country coverage	Sigma (median)
This work	System of Equations	BACI	HS6	Belgium	4.86
Broda and Weinstein (2006)	System of Equations	NBER	HTS10 and SITC-5	US	2.7 - 3.7
Soderbery (2015)	System of Equations	US imports data	HS8	US	1.86
Corbo and Osbat (2013)	System of Equations	COMEXT	ISIC-4	EU27	3.2
Soderbery (2018)	System of Equations	BACI	HS4	192 countries	2.66 – 3.05
Mohler and Seitz (2009)	System of Equations	EUROSTAT	HS8	EU27	4.10
Hertel et al. (2007)	Gravity-type model	Hummels (1999)	5-digit SITC	Argentina, Brazil, Chile, Paraguay, Uruguay, US, and New Zealand	1.8 – 10.1
Fontagné et al. (2022)	Gravity-type model	BACI	HS6	200 countries, including BE	5

Notes: 1) Nomenclatures HTS: Harmonized Tariff Schedule; HS: Harmonized System; SITC: Standard International Trade Classification; ISIC: International Industrial Industry Classification. The number following each nomenclature designates the degree of disaggregation.

2) For highly disaggregated data, we report the median or the mean when the median is not available.

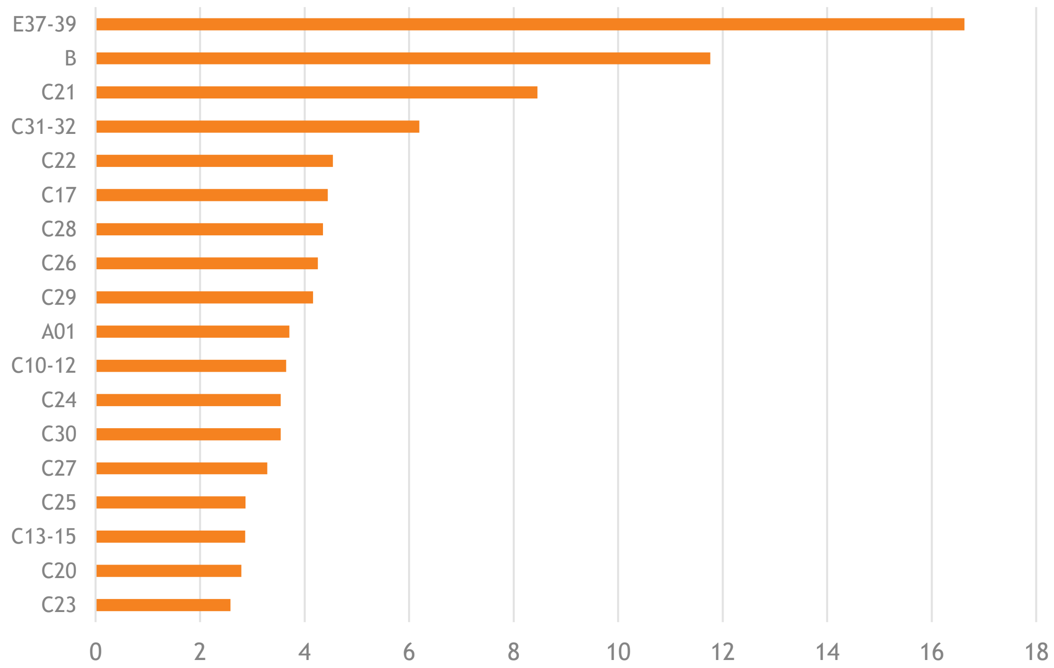
5. (More) Aggregate Elasticities

The second goal of this working paper is to rely on the estimated elasticities to calibrate macroeconomic models. With that goal in mind, this section discusses the procedure to obtain aggregate elasticities. By aggregate elasticities, we mean two things: elasticities for a multi-sector model such as DynEMItE (in Subsection 5.1) and an elasticity for a one sector model such as QUEST III R&D (in Subsection 5.2).

5.1. Sectoral Elasticities for a Multi-sector Model

To obtain "sectoral"[19] elasticities, we estimate Equation (10) with data that are aggregated at the level of CPA A64 products. Though the results are obtained for all product categories that are matched with at least one HS 6-digit product (see Table 10 in the Appendix), we exclude certain product groups for the presentation here. This concerns product groups for which the share in total imports is less than 1% since the measurement error might be tremendous for these groups. This leaves us with 18 CPA product groups in what follows. Graph 4 reports the results for those product groups.

Graph 4 Estimated Armington elasticities, CPA product groups



Note: This graph plots the sectoral Armington elasticities, while the sector consists of a CPA product group, for selected sectors.

The sectoral Armington elasticity ranges from 2.40 to 16.63. Among the 18 CPA product groups retained for the analysis, only 4 of them have an estimated value of the Armington elasticity higher than 5. These product groups are *Furniture and other manufactured goods* (C31-32) with σ equals to 5.87, *Basic pharmaceutical products and pharmaceutical preparations* (C21) with σ equals to 8.45, *Mining and quarrying* (B) with σ equals to 11.76, and *Sewerage services; sewage sludge; waste collection, treatment and disposal services;…* (E37-39) with σ equals to 16.63. At the other extreme, with values lower than 3, the lowest elasticities

[19] In this section, a sector refers to a CPA product group.

are obtained in *Other non-metallic mineral products* (C23), *Chemical and chemical products* (C20), and *Textiles, wearing apparel, leather and related products* (C13-15).

While we found an average of 6.34 and a median of 4.86 at the level of HS 6-digit products, the average and median of the level of CPA product groups are 4.93 and 3.92 respectively. Hence, the elasticity tends to decrease with the level of aggregation, a result commonly found in the literature. Broda and Weinstein (2006) attributed it to the fact that an increase in the level of disaggregation leads to varieties that are more substitutable. For Imbs et Méjean (2015), as discussed in the next subsection, it comes from a systematic bias introduced by aggregate data. Besides, the measurement error is important for CPA product groups as well. In fact, the constant term is significant at all conventional values for all CPA product groups.

Giri, Yi, and Yilmazkuday (2021), Caliendo and Parro (2015) and Ossa (2014) have estimated Armington elasticities at a level of aggregation comparable to the CPA product groups used in this subsection[20]. But one should be cautious when comparing our results to theirs, especially for individual sectors, because of differences in the definition of sectors. In fact, this paper uses the CPA classification which categorizes products that have common characteristics. By contrast, the above-mentioned papers relied on the ISIC classification, which is a classification of all economic activities. Because of this difference in statistical classifications, an accurate mapping between sectors is not possible. In addition, as before, the estimation methodology, the sample and the period covered vary across studies. Nonetheless, these studies provide a basis for comparing our results. Table 11 in Appendix A summarizes the comparison.

Giri, Yi and Yilmazkuday (2021) estimate the Armington elasticities with the simulated method of moments (SMM) applied to 12 OECD countries (including Belgium). Their estimates of the Armington elasticity across 19 ISIC Revision 2 sectors range from 3.97 to 9.94 with a median value of 5.38 and an average of 5.51. Hence, their range is slightly narrower, their average and their median are both slightly higher. For individual sectors, they found an elasticity of 4.57 for *Food products, beverage and tobacco* (ISIC codes = 311, 313 and 314) while we found 3.65 for *Food, beverage and tobacco (CPA code = C10-12).* For *Rubber products,* they came to 5.38 while we found 4.54. For *Transport Equipment,* their estimate is 5.47 while we find 4.16. They also found 3.97 for *Paper and products and printing and publishing* while our estimates are 4.44 and 5.87 respectively for *Paper and paper products* and *Printing and recording services.*

Caliendo and Parro (2015) estimate trade elasticity with a tripled difference technique applied to a gravity equation over 20 (ISIC Revision 3) sectors. Their estimates range from 1.37 to 52.08. The median and the average of their estimates are respectively 7.32 and 9.59. Hence, their range is wider, and their average and median are both higher than those obtained in this paper. For individual sectors, they found 3.55 for *Food* while we found 3.64. They reported 16.72 for *Mining* while we found 11.76 for that sector. For *Plastic*, they found 2.66 while our estimate is 4.54. For *Other transport,* their results indicate an elasticity of 1.37 while we found an elasticity of 2.54. The discrepancy compared to our results is thus small. But this is not always the case. For instance, the estimated elasticity of 9.11 for *Products of agriculture* is significantly higher than ours of 3.71. Likewise, their estimate of *Wood* is 11.83 while we got 2.56.

[20] In HERMES, a macroeconometric model developed at the Federal Planning Bureau (see Bassilière et al. (2013)), the elasticity is also estimated for 6 aggregate industries and found values ranging from 0 to 1.1. These lower estimates compared to ours can be explained by the level of aggregation.

Finally, Ossa (2014) covered 33 (GTAP) sectors and applied the same methodology as in this paper. Their estimates range from 1.91 to 10.07 with an average of 3.42. Their range is slightly narrower, and their average is slightly lower. They found an estimated elasticity of 2.87 for the *Textile* comparable to our estimate of 2.86. For *Other transport equipment,* they found 2.84 while we found 2.54. For *Metal products,* they report 2.70 while we got 2.87. They report 2.34 for *Chemical products* while our results indicate 2.79 for that sector. They found 2.56 for *Paper products* while we obtained 4.44. For *Wood products,* they obtained 2.32 while we obtained 2.56. They obtained 2.20 for *Forestry* while we obtained 2.40. They found 2.92 for *Beverages and Tobacco products* while we found 3.65 for *Food, beverages and tobacco products.* For *Motor vehicles* they report an elasticity of 2.75 while we found 4.16. They found 2.47 for *Other mineral products* while we found 2.58.

Our estimated sectoral elasticities show a strange result for *Coke and refined petroleum products* for which we have found an elasticity of 3.92, which is one of the lowest elasticities. This is rather surprising as this product group is supposed to contain relatively homogenous products and should have one of the highest elasticities. For instance, with a value of 52, it is the sector with the highest elasticity in Caliendo and Parro (2015). In Imbs and Méjean (2010), the elasticity of *Crude petroleum and natural gas* is about 20 and is the third highest elasticity. At the same time, Fontagné et al. (2022) found a value of 4.67 for that product group.

5.2. From Sectoral Elasticities to an Aggregate Elasticity

The Armington elasticities we have estimated so far, either at the HS 6-digit level or across CPA A64 product groups, do not directly match the corresponding elasticity in a one sector model. The reason is that, for a specific product, the estimated trade elasticity, $1-\sigma_k$, measures the relative change in demand from a specific partner $i \neq BE$, to a change in the price of that partner, *ceteris paribus*. That is, the only price that changes in the interpretation is that of the product imported from a unique bilateral partner. The other prices are kept constant. Following Imbs and Méjean (2015), we call this a *micro shock*.

On the contrary, a trade elasticity in a one sector model measures the effect of a shock of aggregate import price on aggregate imports. To estimate it, one may aggregate the data at the level of the economy first, and then estimate Equation (10). However, Imbs and Méjean (2015) proved that doing so would lead to a heterogeneity bias. This results from the fact that, by forcing all sectoral coefficients to be the same, aggregation pushes any existing product level heterogeneity into the residuals, producing a bias when those residuals are correlated with the regressors. The resulting bias turns out to be negative because price changes tend to be large for inelastic products, which can be explained by two reasons.

First, when hit by a cost shock, firms operating under imperfect competition prefer to adjust their markup instead of the price when the products are highly elastic. In contrast, when products are inelastic, they adjust the price. This makes the price of elastic products more stable than those of inelastic products. Second, because the more inelastic the product the higher the distortions caused by tariffs, economic theory implies that high tariffs are imposed on inelastic products. As a result, large changes in prices in aggregate data are associated with low quantity changes, creating an estimate of trade elasticity that is biased towards zero.

To handle this heterogeneity bias, we follow Imbs and Méjean (2015) who proposed a methodology to recover an aggregate elasticity from product-level elasticities. The idea is to map a structural shock to prices in a multi-sector model featuring heterogeneity into a shock in a one-sector model. For the mapping to be useful, however, the two types of shock must be comparable. In a one-sector model, what is measured is the elasticity of aggregate imports. To compare that to a shock in a multi-sector model, Imbs and Méjean (2015) defined a *macro shock* as a shock that affects all relative prices across all sectors uniformly. Thus, in contrast to the micro shock, the macro shock will affect the aggregate price index as well[21].

The macro elasticity, denoted by η_j^M, is defined as the total derivative of total imports in country j[22] with respect to all relative prices when $d\ln\frac{P_{mlj}}{P_{mjj}}\Big|_{P_{mlj}} = 1\ \forall l, m$:

$$\eta_j^M = \sum_m \sum_{l\neq j} \frac{\partial \ln \sum_k \sum_{i\neq j} P_{kij} C_{kij}}{\partial \ln \frac{P_{mlj}}{P_{mjj}}\Big|_{P_{mlj}}} = \sum_k m_{kj} \eta_{kj}^M \tag{11}$$

where $m_{kj} \equiv \frac{\sum_{y\neq j} P_{kyj} C_{kyj}}{\sum_n \sum_{x\neq j} P_{kxj} C_{kxj}}$

η_{kj}^M is in turn defined as the response of sectoral imports of country j. That is, it is the total derivative of total imports of sector k in country j with respect to all relative prices, with $d\ln\frac{P_{mlj}}{P_{mjj}}\Big|_{P_{mlj}} = 1\ \forall l, m$:

$$\eta_{kj}^M = \sum_{l\neq j} \frac{\partial \ln \sum_{i\neq j} P_{kij} C_{kij}}{\partial \ln \frac{P_{kij}}{P_{kjj}}\Big|_{P_{klj}}} \tag{12}$$

Using the optimality conditions implied by the CES structure in (1(1) and (2), one gets:

$$\eta_{kj}^M = (1-\sigma_{kj}) + (1-w_{kjj})(\sigma_{kj}-\gamma_j) + (\gamma_j - 1)\, w_{kj}(1-w_{kjj}) \tag{13}$$

In Equation (13):

- $w_{kjj} \equiv \frac{P_{kjj} C_{kjj}}{\sum_m P_{kmj} C_{kmj}}$ is the share of home (Belgian) products in the total nominal consumption of product k in Belgium.
- $w_{kj} \equiv \frac{P_{kj} C_{kj}}{P_j C_j}$ is the share of product k in the nominal consumption of the home country (Belgium).

Hence, Equation (13) shows how the response of product-level imports to the macro shock (or the sectoral trade elasticity) depends on the Armington elasticity (in nominal terms) and two additional

[21] Recently, Feenstra et al. (2018) have developed a framework in which they distinguish between the macro- and the micro-Armington elasticities. The first governs the substitution between home and foreign goods where the foreign good constitutes a composite of all goods imported from different trading partners. The micro-elasticity, on the contrary, governs the substitution between foreign goods, i.e., the substitution between trading partners. In this framework, the sectoral elasticities we estimate are micro elasticities, and we use the aggregation formula in this section to estimate the macro elasticity.

[22] For clarity, we introduce j, the index of the home country which is Belgium.

elements that reflect the composition of trade. The latter involve the elasticity of substitution between products (γ_j), the share of domestic products in total consumption in Belgium (w_{kjj}), and the share of product k in the total nominal consumption in Belgium (w_{kj}).

Finally, plugging (13) into the definition of the aggregate trade elasticity given by Equation (11), one gets:

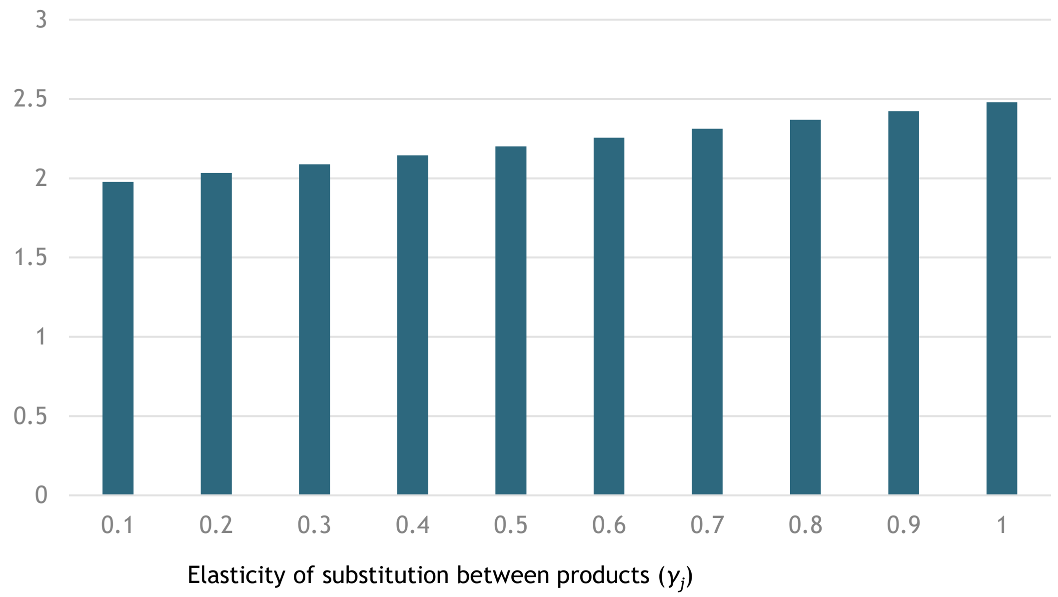
$$\eta_j^M = \sum_k m_{kj} \left((1-\sigma_{kj}) + (\sigma_{kj}-\gamma_j)(1-w_{kjj}) + (\gamma_j - 1)(1-w_{kjj})w_{kj} \right) \tag{14}$$

Equation (14) shows that the aggregate trade elasticity depends on σ_{kj}, m_{kj}, w_{kjj}, w_{kj}, and γ_j. With structural estimates of the first, and calibrated values of the others, it can be computed. In that sense, η_j^M represents a semi-structural estimate of the price elasticity of imports.

To calibrate m_{kj}, w_{kjj}, and w_{kj}, we use data from Supply and Use Tables (SUTs) for 2019. They come from ICN (*Institut des Comptes Nationaux*).

In contrast to m_{kj}, w_{kjj}, w_{kj}, the value of γ_j cannot be observed in data. The only estimates of γ_j we are aware of come from Atalay (2017) who estimates the elasticity of substitution across 30 industries. The results indicate values mostly below 0.2 for the US. Beyond the US, he also considered several countries including Japan and 5 European countries and found an elasticity that always lies below 1. Among the published results for European countries in particular, the elasticity is 0.19 for Italy, 0.28 for Denmark, 0.30 for the Netherlands, 0.36 for Spain, and 0.7 for France. We will make our computation of the aggregate value of the trade elasticity, conditional on the values of γ_j. We consider values of γ_j in line with the results of Atalay (2017) but to make our results comparable to the literature.

Graph 5 Aggregate Armington elasticity for different values of the elasticity of substitution between products (γ_j)



Note: This graph plots the value aggregate Armington elasticity conditional on different values of the elasticity of substitution between CPA product groups.

The aggregate elasticity is plotted on Graph 5 for values of γ_j ranging from 0.1 to 1. It takes values that range from 1.98 to 2.48 and increases with γ_j. When $\gamma_j = 1$ which features the commonly assumed Cobb-Douglas case, the aggregate elasticity is about 2.5. Kastrup et al. (2021), Imbs and Méjean (2010), and Corbo and Osbat (2013) all assumed $\gamma_j = 1$ as well. The estimated aggregate elasticity from Kastrup et al. (2021), who focused on Denmark, ranges from 2.78 and 3.76 depending on the degree of aggregation used for the sectoral elasticities. Imbs and Méjean (2010) report a value of about 2.96 for Belgium with calibrated values of w_{kjj} and w_{kj} computed over the period 1996-2000. Though slightly lower, our estimate of 2.5 compares well to those of both studies. They are, however, well below the aggregate elasticity of 3.7 found by Corbo and Osbat (2013) for Belgium and Luxembourg combined. At the same time, our estimates of the aggregate elasticities are higher than the value of 1.1 used in recent versions of QUEST III R&D for Belgium. It is also higher than 1, the value used in Noname[23], a quarterly one-sector model of Belgium.

23 See Burggraeve & Jeanfils (2008).

6. Conclusion

Despite its relevance in macroeconomic and international trade models, a real consensus on the value of the Armington elasticity is not yet reached. This makes it difficult to rely on the estimated values from the literature for the purpose of calibrating economic models. Moreover, the disaggregation level of empirical studies does not always correspond to that of theoretical models. This working paper estimates the value of the parameter with the objective to calibrate either a one-sector model or a multi-sector model for Belgium.

The estimation of the Armington elasticity is plagued with an endogeneity problem coming from the simultaneous determination of prices and quantities and measurement errors in unit values. To overcome these problems, we follow the methodology developed by Feenstra (1994) which, while taking advantage of the panel structure of trade data, estimates as system of an import supply and an import demand equation. The methodology is further refined to account for theoretically consistent estimates following Soderbery (2015).

Using data from BACI dataset, Armington elasticities are first estimated at the 6-digit HS products. The data are then aggregated at the CPA product groups to estimate the corresponding elasticities. Results point to a strong heterogeneity in the estimates, not only across HS 6-digit products, but also across CPA product groups. For the former, the estimates range from 1.12 to 70.69. For the latter, the estimated elasticities take values between 2.40 to 16.63. When further aggregated at a one-sector level, the value of the Armington elasticity is between 1.98 and 2.48.

The results obtained in this study imply that one should limit the effect of the heterogeneity bias which can affect the predictions of macroeconomic models. That is, the calibration of a multi-sector model should reflect the heterogeneity in Armington elasticities. For a one-sector model, this means that it is desirable to estimate the elasticities with disaggregated data before aggregating them. With this procedure, we found estimates of the aggregate elasticity to be higher than the value used in the last versions of QUEST III R&D for Belgium.

A couple of questions will still need a close attention in future work. First, since services are less recorded in trade databases as the one used in this work, one would like to develop a methodology suited for services. While this seems challenging due to the difficulty in obtaining unit values for services, it will make the analysis of the economy complete. Second, future work could take advantage of the estimates to assess the overall gains to trade in Belgium which could shed more light on how globalization has impacted Belgian consumers.

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Appendix

Appendix A: Additional Tables

Table 9 Simulation results of a 0.5% increase in government investment in QUESTIII R&D, for different values the Armington elasticity

Variables		1 year	3 years	5 years	10 years	20 years
Real GDP	σ = 3	0.48	0.92	1.54	2.95	4.96
	σ = 2	0.48	0.97	1.61	2.97	4.88
Labour produtivity	σ = 3	0.14	0.66	1.26	2.47	4.19
	σ = 2	0.14	0.72	1.33	2.49	4.12
Employment rate	σ = 3	0.24	0.20	0.23	0.39	0.58
	σ = 2	0.25	0.29	0.35	0.49	0.65
Private consumption	σ = 3	0.60	1.04	1.20	1.79	2.93
	σ = 2	-0.20	-0.34	0.35	0.35	1.34
Private investment	σ = 3	-0.04	0.04	0.34	1.20	2.47
	σ = 2	-0.17	-0.21	0.11	1.00	2.22
GDP deflator	σ = 3	0.29	0.04	-0.34	-1.17	-2.30
	σ = 2	0.16	-0.32	-0.82	-1.80	-3.11
Imports	σ = 3	0.91	1.11	0.87	0.49	0.19
	σ = 2	0.50	0.31	0.16	0.07	0.09
Exports	σ = 3	-0.23	0.02	0.45	1.37	2.60
	σ = 2	-0.09	0.40	0.95	2.05	3.52

Table 10 Armington elasticities estimated with the TSLS estimator at the CPA product groups level

Prod NA codes	Prod NA labels	Number of products	Import shares	Adjusted R-squared	σ	Pvalue (σ)	ω
A01	Products of agriculture, hunting and related services	220	3.1%	0.347	3.708	0.000	0.535
A02	Products of forestry, logging and related services	16	0.2%	0.409	2.399	0.000	0.992
A03	Fish and other fishing products; aquaculture products; support services to fishing	5	0.0%	0.729	6.200	0.000	-2.821
B	Mining and quarrying	95	12.3%	0.280	11.764	0.000	3.486
C10-12	Food, beverages and tobacco products	487	7.4%	0.193	3.649	0.000	1.253
C13-15	Textiles, wearing apparel, leather and related products	810	4.0%	0.113	2.861	0.000	1.022
C16	Wood and of products of wood and cork, except furniture; articles of straw and plaiting materials	85	0.7%	0.126	2.546	0.000	0.584
C17	Paper and paper products	126	1.6%	0.814	4.441	0.000	3.604
C18	Printing and recording services	1	0.0%	0.108	5.870	0.000	-135.031
C19	Coke and refined petroleum products	20	0.7%	0.279	3.919	0.001	0.279
C20	Chemicals and chemical products	780	14.4%	0.687	2.788	0.000	1.132
C21	Basic pharmaceutical products and pharmaceutical preparations	100	8.9%	0.477	8.453	0.000	4.175
C22	Rubber and plastic products	136	2.9%	0.191	4.542	0.000	1.856
C23	Other non-metallic mineral products	165	1.1%	0.093	2.584	0.000	1.615
C24	Basic metals	354	5.6%	0.153	3.541	0.000	3.862
C25	Fabricated metal products, except machinery and equipment	244	2.2%	0.240	2.867	0.000	1.511
C26	Computer, electronic and optical products	265	4.70%	0.396	4.256	0.000	4.671
C27	Electrical equipment	186	2.8%	0.104	3.287	0.000	5.744
C28	Machinery and equipment n.e.c.	467	7.0%	0.848	4.350	0.000	2.145
C29	Motor vehicles, trailers and semi-trailers	64	10.7%	0.180	4.161	0.000	2.167
C30	Other transport equipment	84	1.4%	0.486	3.541	0.000	6.131
C31-32	Furniture and other manufactured goods	193	5.3%		6.197	0.000	-6.058
D	Electricity, gas, steam and air conditioning	2	0.3%	NA	NA	NA	NA
E37-39	Sewerage services; sewage sludge; waste collection, treatment and disposal services; materials recovery services; remediation services and other waste management services	74	2.0%	0.554	16.630	0.009	3.616
J58	Publishing services	21	0.3%	0.470	3.935	0.000	4.957
J59-60	Motion picture, video and television programme production services, sound recording and music publishing; programming and broadcasting services	8	0.2%	0.454	14.267	0.006	45.670
M71	Architectural and engineering services; technical testing and analysis services	1	0.0%	NA	NA	NA	3.425
M74-75	Other professional, scientific and technical services and veterinary services	2	0.0%	0.027	3.477	0.065	0.496
R90-92	Creative, arts, entertainment, library, archive, museum, other cultural services; gambling and betting services	7	0.1%	0.167	4.847	4.847	2.303
S96	Other personal services	1	0.0%	0.221	2.334	2.334	2.248
	Total	5,019	1				

Note: The elasticities in this table are obtained with data aggregated at the level of CPA product groups for groups that have matched with at least one 6-digit HS products. NA = not a number.

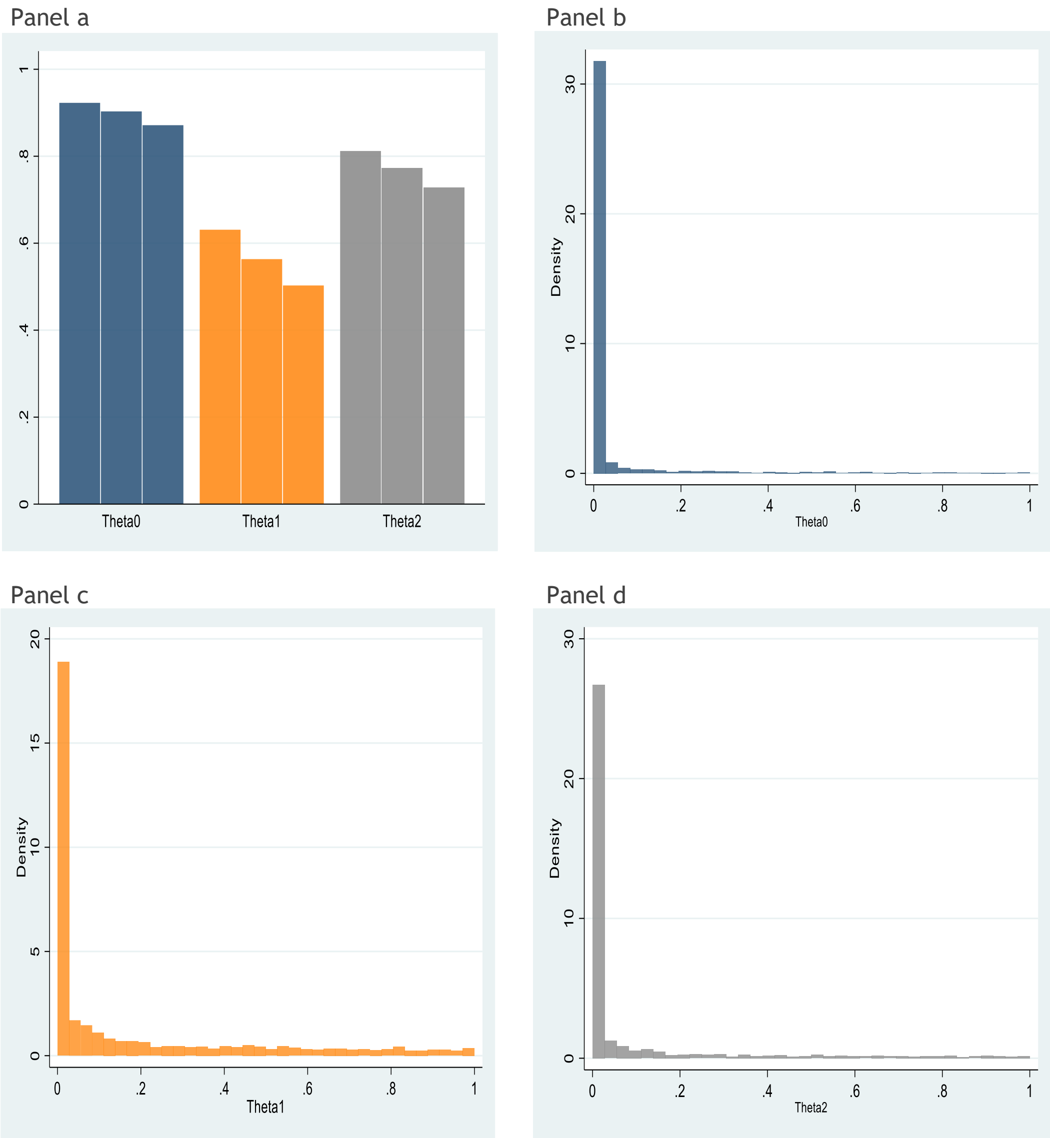
Table 11 Comparison of sectoral elasticities to the literature

	This work	Giri et al. (2021)	Caliendo and Parro (2015)	Ossa (2014)
Products of agriculture, hunting and related services	(A01) 3.71		(1-5) 9.11	
Products of forestry, logging and related services	(A02) 2.40			(frs) 2.20
Mining and quarrying	(B) 11.76		(10-14) 16.72	
Food, beverages and tobacco products	(C10-12) 3.65	(311) 4.57	(15-16) 3.55	
	(C10-12) 3.65	(313, 314) 4.57		(b_t) 2.92
Textiles, wearing apparel, leather and related products	(C13-15) 2.86	(321) 4.27	(17-19) 6.56	(tex) 2.87
	(C13-15) 2.86	(322) 5.41		(wap) 5.39
	(C13-15) 2.86	(323) 6.28		(lea) 3.67
Wood and of products of wood and cork, except furniture; articles of straw and plaiting materials	(C16) 2.55	(331) 5.17	(20) 11.83	(lum) 2.32
Furniture and other manufactured goods	(C31-32) 6.20	(332) 5.47		
Paper and products	(C17) 4.44			
Printing and recording services	(C18) 5.87	(341, 342) 3.97	(21-22) 10.07	(ppp) 2.56
Coke and refined petroleum	(C19) 3.92		(23) 52.08	
Chemicals and chemical products	(C20) 2.79		(24) 5.75	(chm) 2.34
Rubber and plastic products	(C22) 4.54	(355) 5.38		
	(C22) 4.54	(3.87) 4.87	(25) 2.66	
Other non-metallic mineral products	(C23) 2.58	(369) 4.87	(27) 7.99	(nmm) 2.47
Fabricated metal products, except machinery and equipment	(C25) 2.87	(381) 6.07	(28) 5.30	
Electrical equipment	(C27) 3.29		(31) 11.60	
Machinery and equipment n.e.c.	(C28) 4.35	(383) 4.27	(29) 2.52	
Motor vehicles, trailers and semi-trailers	(C29) 4.16	(384) 5.47	(34) 2.01	(mvh) 2.75
Other transport equipment	(C30) 3.54		(351 – 359) 1.37	(otn) 2.84
Estimation methodology	System of Equations	Gravity-type Equations	Gravity-type Equations	System of Equations
Classification	CPA products	ISIC Revision 2	ISIC Revision 3	GTAP 8
Number of sectors covered	30	19	20	33
Mean	4.23	5.51	5.55	3.42
Range		3.97 – 9.94	1.37 – 52.08	1.91 – 10.07

Note: In parentheses are the sector codes associated with the classification used by the study.

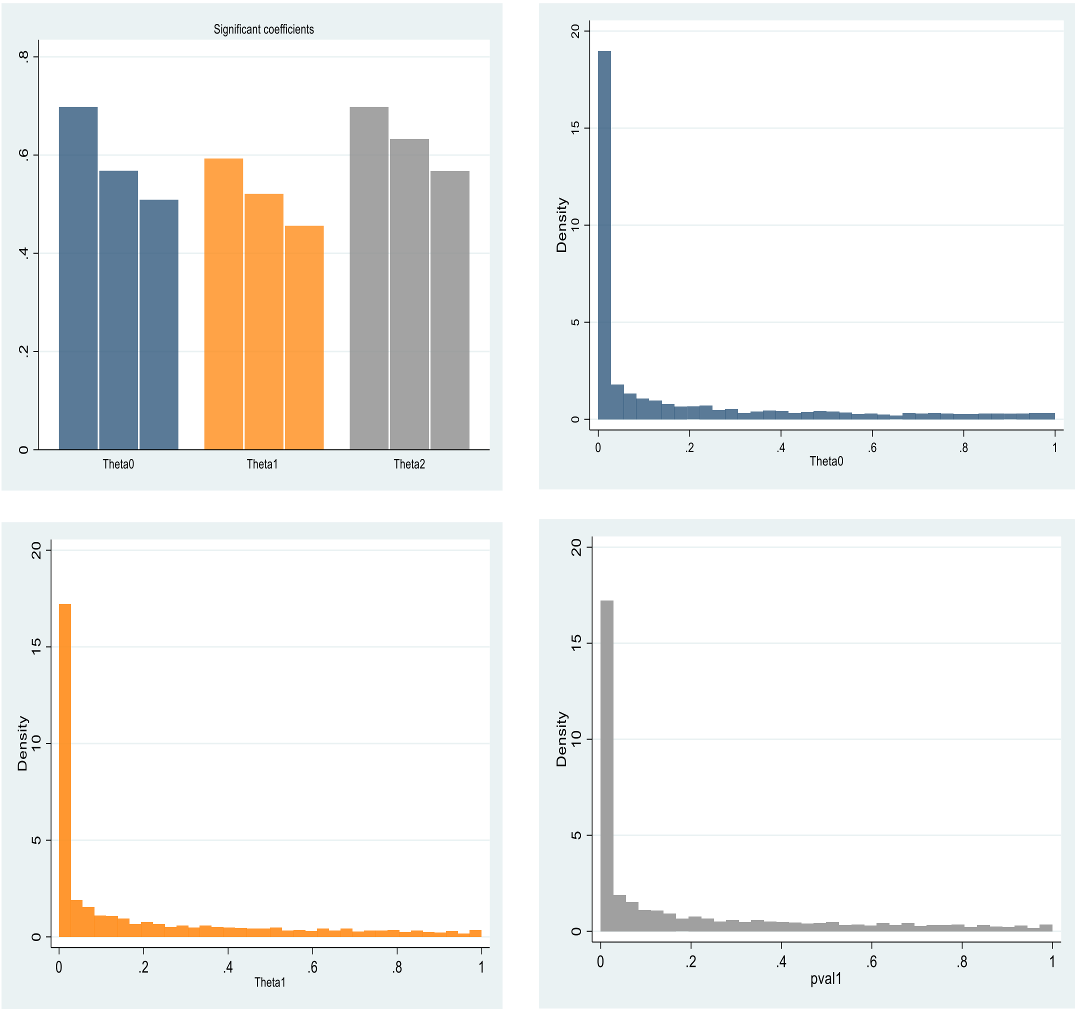
Appendix B: Additional Graphs

Graph 6 Share of significant reduced-form coefficients and their p-value from the TSLS estimation



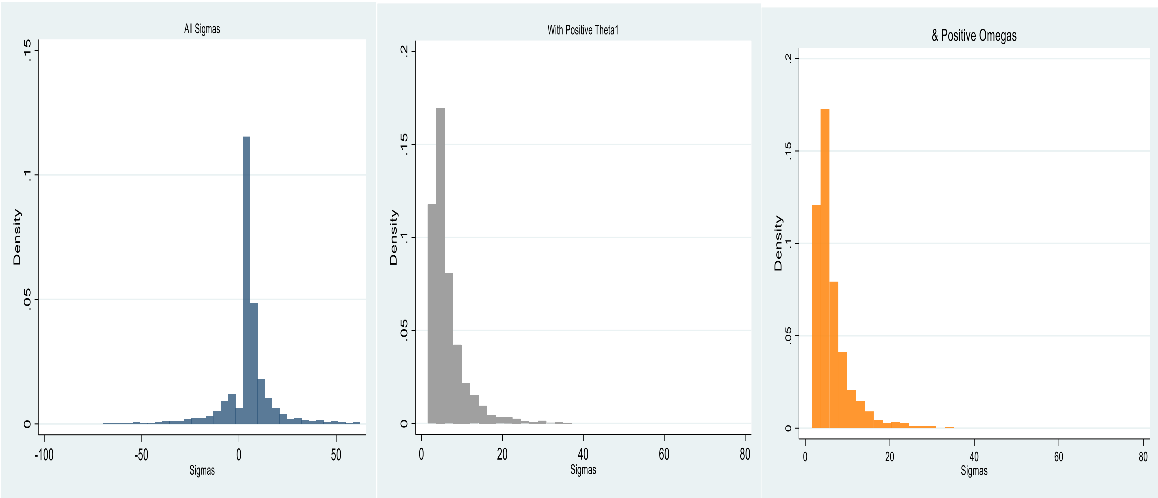
Source: The results are derived from the estimates in Table 5.
Note: For each reduced-form coefficient, we compute the share of estimates that are significant respectively at the 10%, 5% and 1% (Panel a). The p-value of the coefficients are shown on Panels b-d.

Graph 7 Share of significant reduced-form coefficients and their p-value from the LIML estimation



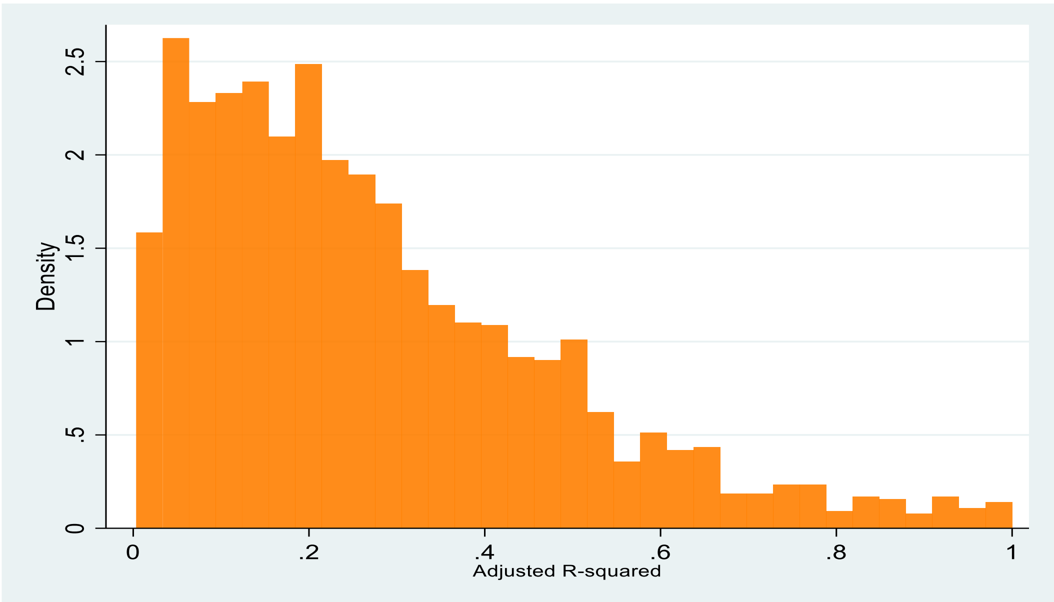
Source: The results are derived from the estimates in Table 6.
Note: For each reduced-form coefficient, we compute the share of estimates that are significant respectively at the 10%, 5% and 1% (Panel a). The p-value of the coefficients are shown on Panels b-d.

Graph 8 Histograms of the distribution of the Armington elasticities



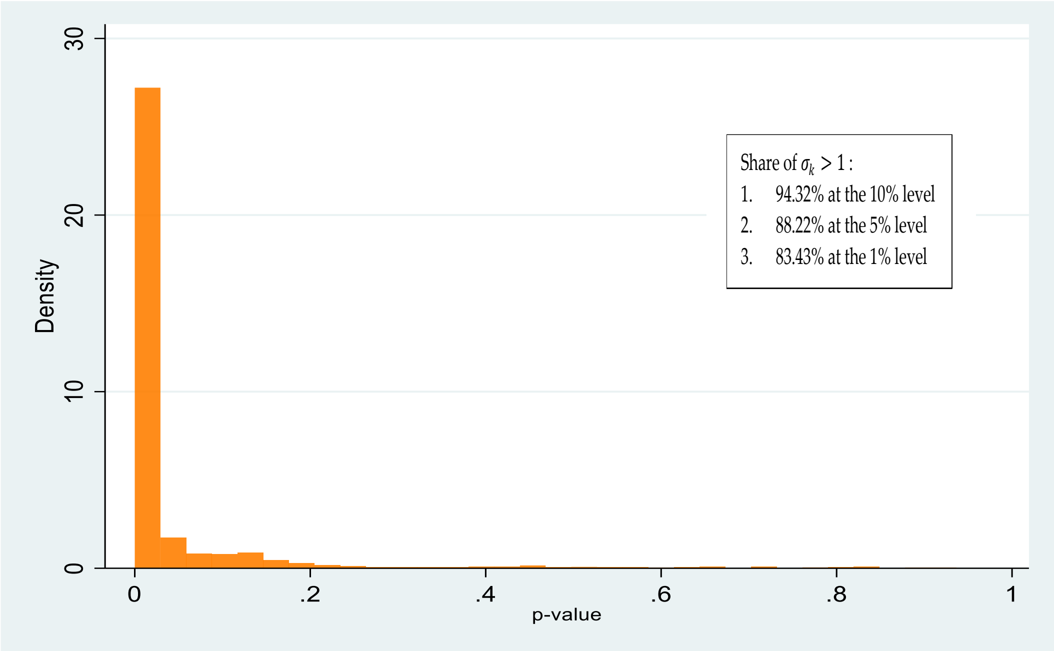
Source: Estimates corresponding to Panel B of Table 5. “All” corresponds to Row (1), “Positive theta1” corresponds to Row (2) and “Positive omega corresponds to Row (3).
Note: For the sake of exposition, the raw estimates, those labelled “All” are trimmed at the 5% level.

Graph 9 Adjusted R-squared of the preferred results



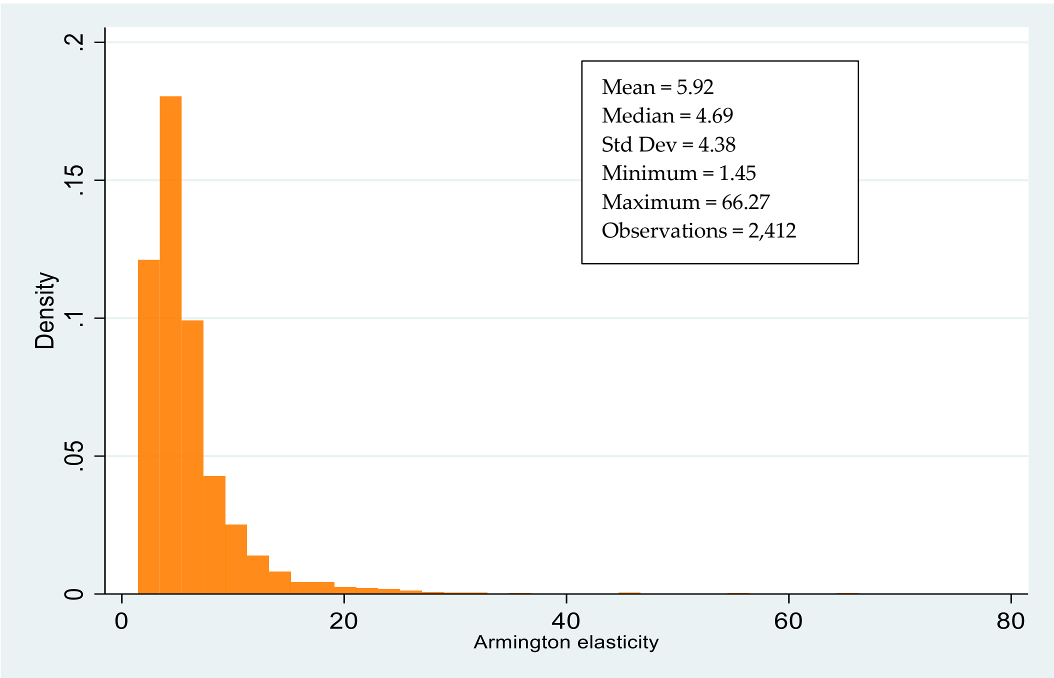
Note: This graph shows the adjusted r-squared associated with the results in Panel B of Table 5.

Graph 10 Histograms of the distribution of p-value of the estimated Armington elasticities



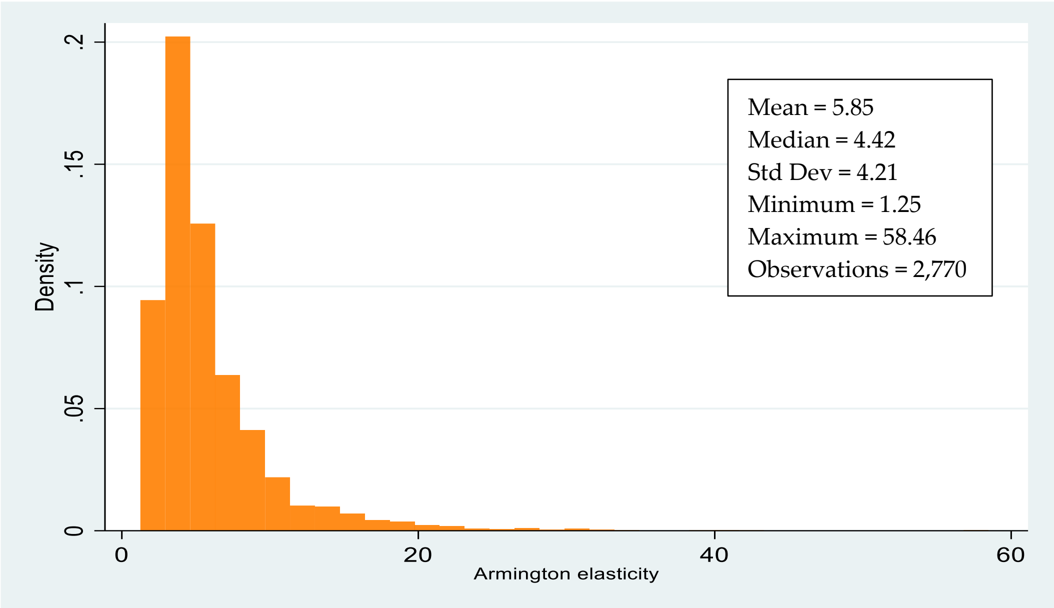
Note: These are the p-values associated with Row (3) of Panel B in Table 5. The standard errors are computed with the Delta method.

Graph 11 Armington elasticity with the second largest supplier as the reference country



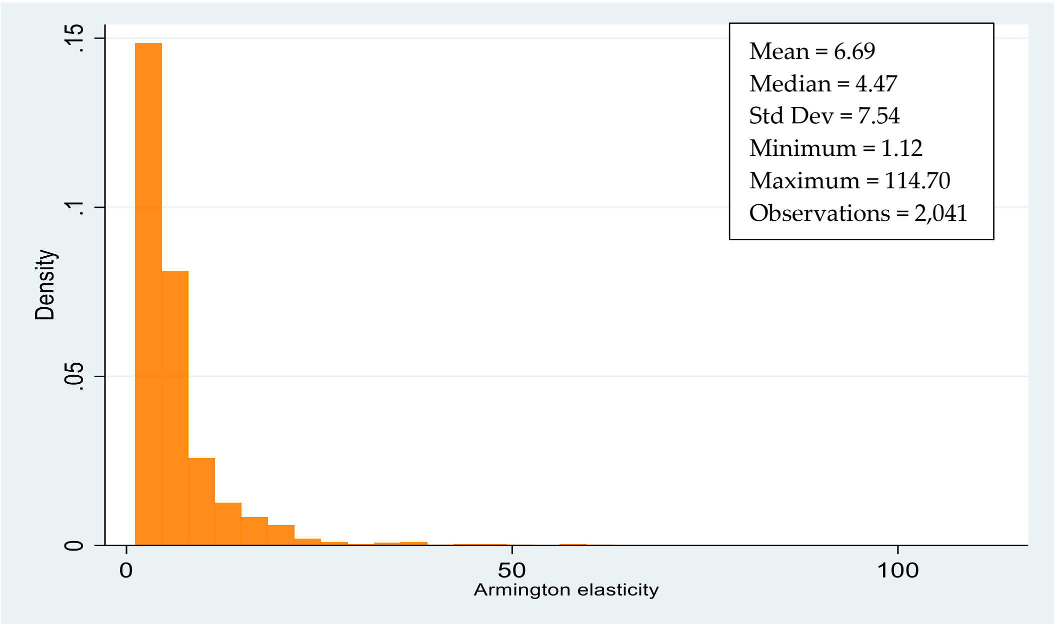
Note: This graph is produced with Armington elasticities obtained when the reference country is the second largest supplier.

Graph 12 Armington elasticity with the HS2 version of the BACI dataset



Note: This graph is produced with Armington elasticities obtained with the HS2 version of the dataset.

Graph 13 Armington elasticity constraint on θ_1



Note: This graph is produced with Armington elasticities obtained with the HS2 version of the dataset.

Appendix C: Recovering the Armington elasticity from the reduced-form coefficients

As a reminder, the estimated regression reads:

$$Y_{kit} = \theta_{1k} X_{1kit} + \theta_{2k} X_{2kit} + u_{kit}$$

where: $\theta_{1k} = \frac{\omega_k}{(1+\omega_k)(\sigma_k - 1)}$ *(a)* and $\theta_{2k} = \frac{1 - \omega_k(\sigma_k - 2)}{(1+\omega_k)(1-\sigma_k)}$ *(b)*

To solve for the structural parameters, Feenstra (1994) defined a new parameter ρ_{kj} as follows:

$$\rho_k = \frac{\omega_k(\sigma_k - 1)}{1 + \omega_k \sigma_k}$$

so that $0 \leq \rho_k < \frac{(\sigma_k - 1)}{\sigma_k} < 1$. These restrictions serve to obtain theoretically consistent estimates of σ_k and ω_k. ρ_k is interpreted as the correlation between the vertical shift in the demand and the change in the equilibrium price. When ω_k is also equal to zero, the value of ρ_k is equal to zero as well.

Using the definition of ρ_k, θ_{1k} and θ_{2k} become:

$\theta_{1k} = \frac{\rho_k}{(1-\rho_k)(\sigma_k - 1)^2}$ (c) and $\theta_{2kj} = \frac{(2\rho_k - 1)}{(1-\rho_k)(\sigma_k - 1)}$ (d)

Using the estimated values of θ_{1k} and θ_{2k} coupled with the *(c)* and *(d)*, the estimated values of σ_k and ρ_k (and thus ω_k) can be recovered. The following proposition in Feenstra (1994)[24], allows to recover the value of the structural parameters.

Proposition: So long as $\hat{\theta}_{1k} > 0$, then:

- *If $\hat{\theta}_{2k} > 0$ then:*

$$\hat{\rho}_k = \frac{1}{2} + \left(\frac{1}{4} - \frac{1}{4 + \left(\frac{\hat{\theta}_{2k}^2}{\hat{\theta}_{1k}} \right)} \right)^{1/2}$$

- *If $\hat{\theta}_{2k} < 0$ then*

$$\hat{\rho}_k = \frac{1}{2} - \left(\frac{1}{4} - \frac{1}{4 + \left(\frac{\hat{\theta}_{2k}^2}{\hat{\theta}_{1k}} \right)} \right)^{1/2}$$

and in either case,

$$\hat{\sigma}_k = 1 + \left(\frac{2\hat{\rho}_k - 1}{1 - \hat{\rho}_k} \right) \frac{1}{\hat{\theta}_{2k}} > 1$$

[24] For the notation, if x designates the true value of a parameter, then $\hat{x}$ designates the estimate of the parameter.

The above proposition for obtaining economically consistent estimates of $\hat{\sigma}_{kj}$ holds only when $\hat{\theta}_{1k} > 0$. But it fails when $\hat{\theta}_{1k}$ is so negative that imaginary or economically infeasible values of $\hat{\sigma}_k$ and $\hat{\rho}_k$ are obtained.

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