WORKING PAPER

8-00

The NIME Model

Specification and Estimation of the Demand Equations of the Household Sector

Federal
Planning Bureau
Economic analyses and forecasts

Avenue des Arts 47-49 Kunstlaan B-1000 Brussels
Tel.: 02 507 73 11
Fax: 02 507 73 73
E-mail: contact@plan.be
URL: http://www.plan.be

Eric Meyermans and
Patrick Van Brusselen

October 2000

The NIME Model

Specification and Estimation of the
Demand Equations of the
Household Sector

Eric Meyermans and
Patrick Van Brusselen

October 2000

Federal Planning Bureau

The Belgian Federal Planning Bureau (BFPB) is a public agency under the authority of the Prime Minister and the Minister of Economic Affairs. The BFPB has a legal status that gives it an autonomy and intellectual independence within the Belgian federal public sector.

BFPB's activities are primarily focused on macro-economic forecasting, analysing and assessing policies in the economic, social and environmental fields.

Internet

URL: http://www.plan.be

E-mail: contact@plan.be

Publications

Publications:

The Economic Forecasts
The "Short Term Update"

Planning Papers (the last numbers)

giving the results of policy analyses and assessments

85 *Un projet à l'échelle mondiale: le développement durable*
Nadine Gouzée, Stéphane Willems, Natacha Zuinen - Février 2000

86 *Vieillissement démographique et financement de la sécurité sociale: un défi soutenable?*
(Actes du Colloque organisé par le Bureau fédéral du Plan à Bruxelles, les 2 et 3 décembre 1997)

87 *Les participations publiques dans le secteur marchand en Belgique*
H. Spinnewyn - Février 2000

Working Papers (the last numbers)

4/00 *La simplification des procédures administratives pesant sur les entreprises*
C. Kegels - Mai 2000

5/00 *A description of the HERMES II model for Belgium*
F. Bossier, I. Bracke, P. Stockman, F. Van Horebeek - July 2000

6/00 *Croissance et emploi dans le secteur des services - Une analyse comparative*
M. Van Overbeke - Août 2000

7/00 *Macro-economische en budgettaire effecten van de dollarappreciatie en de duurdere ruwe olie*
F. Bossier en F. Vanhorebeek - September 2000

Citations should refer to the publication, mentioning the author(s) and the date of publication.

Responsible Editor Henri Bogaert
Legal Depot: D/1999/7433/21

Table of Contents

The NIME Model - Specification and Estimation of the Demand Equations of the Household Sector

Eric Meyermans and Patrick Van Brusselen

In 1999, the Belgian Federal Planning Bureau launched a research program to develop an econometric model to analyse the effects of policy variants and other exogenous economic shocks on the Belgian and European economies, taking explicitly into account the euro zone institutional framework and the international economic environment. So far, the Bureau's efforts have led to the construction of a first version of the New International Model for Europe (NIME), of which the different parts will be presented in several working papers.

The current version of NIME divides the world into six separate blocks: the EU block consisting of the countries that joined EMU in January 1999 minus Belgium, the NE block consisting of the EU countries that did not join EMU, the United States, Japan and the rest of the world. The model describing the Belgian economy would consist of the short term or the medium term macroeconomic model currently in use at the Federal Planning Bureau.

The present working paper describes the household sector of NIME. We start by deriving the long run equilibrium plans of the household sector on the basis of an intertemporal optimization problem. The obtained set of demand equations explains the demand for goods, services and assets as a function of the nominal interest rate, the real interest rate, the user cost of residential buildings, and the available means. In the empirical section, we also assume that rigidities prevent households from adjusting their expenditures immediately to their long run equilibrium plans. An error correction mechanism and a partial adjustment scheme are used to capture these rigidities. Finally, estimation results for the household sector of the EU, NE, US and JP blocks are shown.

I Introduction

In the past, the Belgian Federal Planning Bureau (FPB) made intensive use of the HERMES-Link system of macrosectoral econometric models, developed in the eighties by a consortium of European research centres, under the auspices of the European Commission [1]. However, this system, composed of eight large sectoral national models and four bilateral trade flow models, had gradually become outdated since it was much too large to be overhauled by the FPB on its own. Therefore, it was decided in 1999 to develop a new, easier to maintain, international macroeconomic model, that would be more in line with recent developments in econometric theory and practice, as well as with the present goals of the Bureau. So far, the FPB's efforts have led to the construction of a first version of New International Model for Europe (NIME), of which the different parts will be presented in several working papers [2].

The NIME model is an econometric model to analyse the effects of policy variants and other exogenous economic shocks on the Belgian and European economies, taking explicitly into account the euro zone institutional framework and the international economic environment. Similar to other major international macroeconomic models (see, for example, Laxton et al. (1998)), the current version of NIME divides the world into six separate blocks: the EU block consisting of the countries that joined EMU in January 1999 [3] minus Belgium, the NE block consisting of the EU countries that did not join EMU in January 1999 [4], the United States, Japan and the rest of the world. The sixth block, describing the Belgian economy, would consist of the short term or the medium term macroeconomic model currently in use at the FPB (see, for example, Bossier et al. (2000)). These six country blocks are to be linked to each other through trade and financial flows.

In each country block, except for the "rest of the world" block, we distinguish a household sector, an enterprise sector, a fiscal sector, and a monetary sector [5]. A set of behavioural relations and accounting identities is specified for each of these sectors. The long run behavioural relations are derived from an explicit optimization problem, while error correction mechanisms and partial adjustment schemes are used to capture sluggish adjustment to these long run plans.

---

1. For a description of the HERMES-Link model, see Commission of the European Communities (1993).
2. Comments on these working papers are welcome and should be mailed to Eric Meyermans at em@plan.be or Patrick Van Brusselen at pvb@plan.be .
3. The ten EU block countries are : Austria, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain.
4. The four NE block countries are : Denmark, Greece, Sweden, and the United Kingdom.
5. The "rest of the world" block will consist of only a very limited number of equations describing international trade.

This paper presents the household sector of the NIME model. The other parts of the model will be presented in future working papers. In the second section of this paper, we derive the long run equilibrium plans of the household sector on the basis of an intertemporal optimization problem. The obtained set of demand equations explains the demand for goods, services and assets as a function of the nominal interest rate, the real interest rate, the user cost of residential buildings, and the available means.

In the third section we present some empirical results. In the empirical section we assume that rigidities prevent households from adjusting their expenditures immediately to their long run equilibrium plans. An error correction mechanism and a partial adjustment scheme are used to capture these sluggish adjustment processes. Estimation results for the EU, NE, US and JP blocks are shown.

II Specification of a Set of Demand Equations for the Household Sector

The NIME model consists of six blocks. For each block, we assume that there exists a single representative agent for the whole household sector. First, we specify the intertemporal utility function and the intertemporal budget constraint of this representative economic agent. Next, we derive a set of differentiable demand equations by assuming that the household sector maximizes its intertemporal utility, subject to the intertemporal budget constraint and a set of predetermined prices. This set of equations explains the demand for goods and services as a function of, i.a., the available means, the nominal interest rate, the user cost of residential buildings, and the real interest rate [1].

A. The intertemporal allocation problem of the household sector

In the first subsection we determine the available means of the household sector. The available means of the household sector consists of assets inherited from the past, the return on these assets, and labour income. In the second subsection we specify a differentiable intertemporal utility function for the household sector. This function is defined over a field of contemporaneous and future goods and services. In the third subsection we formulate the optimization problem that defines how the household sector allocates its available means between the consumption of goods, residential buildings, money, and other assets.

1. The intertemporal budget constraint

The total available means of the household sector in period t is equal to [2]:

$$(1)\ WHU_t = CIRO_{t-1}\,(1-\rho_{iro})\ PCIR_t + INVHO_{t-1}\,(1-\rho_{invh})\ PINVH_t$$
$$+ M_{t-1} + CAOU_{t-1}\,(1+LIC_{t-1}) + WRP_t\,NP_t + WRG_t\,NG_t + UB_t\,(LS_t - (NP_t + NG_t))$$
$$+ TRANS\_00_t + NOIH_t - DTH_t - DTC_t$$

1. See, for example, Laxton et al. (1998), Powell and Murphy (1997), Roeger and in 't Veld (1997), or Brayton and Tinsley (eds.) (1996), for the treatment of the household sector in other macroeconomic models.
2. For notational convenience, we do not use labels to indicate a block. It is assumed that the structure of the different blocks is similar. Mutatis mutandis, the analytical results obtained in this section apply to all blocks.

with:

$CAOU_t$: net other assets, in current prices,
$CIRO_t$: stock of residential buildings, in constant prices,
DTC_t : taxes on asset income, in current prices,
DTH_t : taxes on labour income, in current prices,
$INVHO_t$: stock of inventories, in constant prices,
LIC_t : domestic interest rate,
LS_t : total labour supply,
M_t : nominal money stock,
NG_t : employment in the public sector,
$NOIH_t$: net other income of households, in current prices,
NP_t : employment in private sector,
$PCIR_t$: price of residential buildings,
$PINVH_t$: price of household inventories,
$TRANS\_00_t$: other net transfers of government to households, in current prices,
UB_t : unemployment benefits, in current prices,
WHU_t : total available means of household sector, in current prices,
WRG_t : wage in the public sector, in current prices,
WRP_t : wage in the private sector, in current prices,
ρ_{iro} : rate of depreciation of the stock of residential buildings,
ρ_{invh} : rate of depreciation of inventories.

In other words, equation (1) states that the total available means of the households are equal to the stock of assets inherited from the past, plus the income generated by these assets and by the supply of labour. Households also receive transfers from the government, and they pay direct taxes on labour and asset income. Labour is employed either in the private or in the public sector, where it earns a private sector or a public sector wage, respectively. If one is unemployed one receives an unemployment benefit [1].

The household sector spends its available means on the consumption of commodities, money, residential buildings and other assets, i.e. the expenditures of the household sector in period *t* are equal to:

(2) $$EXHU_t = CPO_t\, PCH_t + M_t + CIRO_t\, PCIR_t + INVHO_t\, PINVH_t + CAOU_t$$

with:

CPO_t: consumption of goods and services (other than monetary services and services generated by residential buildings), in constant prices,

$EXHU_t$: total expenditures of the household sector, in current prices,

PCH_t: consumer price index.

In each time period *t* the budget constraint has to be met, i.e.:

(3.a) $$EXHU_t = WHU_t\, .$$

1. It should be noted that, in this paper, income from labour is assumed to be predetermined. The detail of the sources of labour income is used here to facilitate future reference, when we present a paper analysing the labour market in NIME.

Inserting equation (1) and equation (2) into equation (3.a) yields:

$$\text{(3.b) } CIRO_{t-1}\,(1-\rho_{iro})\; PCIR_t + INVHO_{t-1}\,(1-\rho_{invh})\; PINVH_t + M_{t-1}$$
$$+ CAOU_{t-1}\,(1 + LIC_{t-1}) + WRP_t\, NP_t + WRG_t\, NG_t + UB_t\,(LS_t - (NP_t + NG_t))$$
$$+ TRANS\_00_t + NOIH_t - DTH_t - DTC_t = CPO_t\, PCH_t + M_t + CIRO_t\, PCIR_t$$
$$+ INVHO_t\, PINVH_t + CAOU_t$$

or, on rearranging terms:

$$\text{(3.c) } (CIRO_t - CIRO_{t-1}(1-\rho_{iro}))\, PCIR_t + (CAOU_t - CAOU_{t-1}) + (M_t - M_{t-1})$$
$$+ PINVH_t\,(INVHO_t - INVHO_{t-1}(1-\rho_{invh})) + CPO_t\, PCH_t = WRP_t\, NP_t$$
$$+ WRG_t\, NG_t + UB_t\,(LS_t - (NP_t + NG_t)) + TRANS\_00_t + NOIH_t - DTH_t - DTC_t$$
$$+ CAOU_{t-1}\, LIC_{t-1}$$

which is the intertemporal budget constraint of the household sector.

Equation (3.c) indicates that the change in the money stock, in the stock of residential buildings, and in the other assets, plus contemporaneous consumption has to be equal to net labour income plus income from the assets.

2. The intertemporal utility function

In period *t* the representative economic agent will continue to live during *T-t* periods, i.e. his planning horizon reaches until period *T*. In period *T* he consumes his last bundle of commodities and services, and he leaves a bequest, Z_T.

For each period in time the economic agent can formulate the utility which some convex combination of commodities and services render, and these combinations can be compared with each other in utility terms. Formally speaking, the intertemporal utility calculus of the household sector is described by a well behaved, twice differentiable continuous intertemporal utility function which is strongly quasi-concave and runs as follows:

$$\text{(4)} \quad V(\,(CPO_t, MS_t, WS_t, L_t), \ldots, (CPO_k, MS_k, WS_k, L_k), \ldots, (CPO_T, MS_T, WS_T, L_T), Z_T\,)$$

with:

L_t: leisure,[1]

MS_t: monetary services,

T: planning horizon,

WS_t: services generated by residential buildings,

1. Note that $L_t = A - LS_t$, with A defined as total available time. In other words, the labour supply and leisure are each others' mirror image.

and where Z_T is the expected future purchasing power in period *T+1*, generated by the portfolio of assets held at the end of the planning horizon, i.e.:

$$Z_T = (M_T + CIRO_T(1 - \rho_{iro})\,PCIR_{T+1} + INVHO_T\,(1 - \rho_{invh})\,PINVH_{T+1} + CAOU_T\,(1 + LIC_T) + ZY_{T+1})\ /\ PCH_{T+1}$$

with ZY_{T+1} defined as the non-asset income accruing in period *T+1*.

Monetary services and services generated by residential buildings cannot be observed directly. We assume that monetary services, MS, are a function of the available stock of real currency balances conditional on the amount of purchases of consumer goods (see, for example, Patinkin (1989)), i.e.:

(5.a) $$MS_t = MS\,(M_t / PCH_t;\ CPO_t)$$

with MS(.) a continuous, twice differentiable function.

We also assume that the services generated by residential buildings are a function of the available stock of residential buildings, i.e.:

(5.b) $$WS_t = WS\,(CIRO_t)$$

with WS(.) a continuous, twice differentiable function.

Inserting equations (5.a) and (5.b) into equation (4) yields the intertemporal, continuous, twice differentiable utility function:

(6) $$U\,(\,(CPO_t, M_t / PCH_t, CIRO_t, L_t)\,, ..., (CPO_k, M_k / PCH_k, CIRO_k, L_k), ..., (CPO_T, M_T / PCH_T, CIRO_T, L_T), Z_T\,)\ .$$

A special case is obtained if we assume that *t = T*, so that the utility function (6) can be rewritten as [1]:

(7) $$U\,(CPO_t, M_t / PCH_t, CIRO_t, L_t, Z_t)\ .$$

1. Note that a similar result can be obtained by recursive programming when T >t. Making use of recursive programming we derive from equation (6) the direct-indirect utility function: $U\,(CPO_t, M_t / PCH_t, CIRO_t, L_t, Z_t\,;\,PCH_{t+1}, ..., PCH_T\,;\,PCIR_{t+1}, ..., PCIR_T\,;\,LIC_{t+1}, ..., LIC_T\,;\,ZY_{t+1}, ..., ZY_T)$. The difference between this equation and equation (7) is that in the former equation, the utility of a bundle of commodities consumed at moment t is conditional on future prices and income, which is not the case for equation (7). In empirical terms, this implies that if we follow the strategy outlined in the main text, then we do not have to compile a databank consisting of future prices and non-asset income.

Further a priori structure can be given to utility function (7) by assuming separability between the decision to consume goods and services, on the one hand, and the decision to take leisure, on the other hand:

$$(8) \qquad U\,(\,U1(CPO_t, M_t / PCH_t, CIRO_t, Z_t), U2(L_t)\,)\,.$$

In other words, the sub-utility of a convex combination of commodities and services does not depend on the amount of leisure, and vice-versa. The preference ordering specified in equation (8) allows us to study the decisions related to the consumption of goods and services separately from the decisions related to the supply of labour. Let us now investigate how the household sector allocates its available means over goods and services [1].

3. The intertemporal optimization problem

Equation (8) describes the intertemporal utility function of the household sector, while equation (3.c) describes the intertemporal budget constraint. Under separability, the optimization problem for the expenditures of the household sector can now be written as [2]:

$$(9) \qquad \underset{CPO_t,\, M_t,\, CIRO_t,\, CAOU_t,\, INVHO_t}{MAX} \; U1\,(CPO_t, M_t / PCH_t, CIRO_t, Z_t)$$

subject to:

$$CIRO_{t-1}\,(1-\rho_{iro})\;PCIR_t + INVHO_{t-1}\,(1-\rho_{invh})\;PINVH_t + M_{t-1}$$
$$+ CAOU_{t-1}\,(1+LIC_{t-1}) + WRP_t\,NP_t + WRG_t\,NG_t + UB_t\,(LS_t - (NP_t + NG_t))$$
$$+ TRANS\_00_t + NOIH_t - DTH_t - DTC_t = CPO_t\,PCH_t + M_t + CIRO_t\,PCIR_t$$
$$+ INVHO_t\,PINVH_t + CAOU_t$$

with Z_t defined as:

$$Z_t = (M_t + CIRO_t(1-\rho_{iro})\,PCIR_{t+1} + INVHO_t\,(1-\rho_{invh})\,PINVH_{t+1}$$
$$+ CAOU_t\,(1+LIC_t) + ZY_{t+1}) / PCH_{t+1}\,.$$

The first order conditions for an optimum of this problem are derived in Appendix A. These first order conditions can be summarized by defining the following vector of goods, Y_t', and prices, Π'_t:

$$(10.a) \qquad Y_t' = (CPO_t, \frac{M_t}{PCH_t}, CIRO_t, Z_t)$$

1. See Deaton and Muellbauer (1987) for more details regarding separability.
2. Here we assume that the prices are predetermined.

and

$$(10.b) \qquad \Pi'_t = \left(PCH_t, \frac{LIC_t}{(1+LIC_t)} PCH_t, PCIR_t - \frac{PCIR_{t+1}(1-\rho_{iro})}{1+LIC_t}, \frac{PCH_{t+1}}{1+LIC_t}\right) \equiv (PCH_t, PM_t, USERIR_t, PZ_t)$$

i.e. the vector Y consists of goods, real money balances, residential buildings, and net other assets, while the vector of prices consists of the corresponding prices and opportunity costs. Here, the price of one unit of CPO_t is equal to PCH_t.

The interpretation of the opportunity cost of money is as follows. In order to hold one unit of real money balances, M_t/PCH_t, one has to spend PCH_t units of the currency. By holding PCH_t units of money instead of an interest-bearing financial asset, one foregoes a yield equal to LIC_t PCH_t. The present value of this is:

$$(11.a) \qquad PM_t = \frac{LIC_t}{1+LIC_t} PCH_t .$$

The interpretation of the user cost of residential buildings is as follows. Buying one unit of housing in period *t* costs $PCIR_t$. Using this house during the period *t* will depreciate its value by ρ_{iro} percent, so that one will get a price equal to $PCIR_{t+1}(1-\rho_{iro})$ when one sells that house in period *t+1*.

The present value in period *t* of the latter is equal to $\frac{PCIR_{t+1}(1-\rho_{iro})}{1+LIC_t}$.

In other words, the user cost of owning the house during one period is equal to:

$$USERIR_t = PCIR_t - \frac{PCIR_{t+1}(1-\rho_{iro})}{1+LIC_t}$$

which can also be rewritten as:

$$(11.b) \qquad USERIR_t = \frac{(1+LIC_t) - \left(\frac{PCIR_{t+1}}{PCIR_t}\right)(1-\rho_{iro})}{1+LIC_t} PCIR_t .$$

Finally, bonds [1] are a means to transfer purchasing power from one period to the other. These bonds have an interest rate equal to LIC_t. The expected purchasing power in period *t+1* of one unit bought in period *t* is equal to $(1+LIC_t)/PCH_{t+1}$. If one wants to obtain one real unit of purchasing power in the next period, by holding bonds, one has to pay today the unit price:

$$(11.c) \qquad PZ_t = \frac{PCH_{t+1}}{1+LIC_t} .$$

1. "Bonds" refers here to all other assets of the household sector.

B. A set of demand equations

We proceed here by postulating a set of demand equations which can be used to estimate the interaction between the quantities, prices, and the available means.

1. Household demand

Consider the following set of log-linear demand equations which relates the demanded quantities of goods and services, Y_t, to the prices Π_t, and to the available means:

$$(12) \quad \ln(Y_t) = y\_l0 + y\_lb \ln(SCALE_t) + y\_l1 \ln(PCH_t) + y\_l2 \ln(PM_t) + y\_l3 \ln(USERIR_t) + y\_l4 \ln(PZ_t) + y\_l5\, H_t$$

for $Y_t = CPO_t, M_t / P_t, CIRO_t, Z_t,$ and with y = cp, m, cir, z .

The scale variable, SCALE, is a measure of total real purchasing power of the household sector. In the empirical section, this variable is approximated by real household disposable income. The variable H_t is short for every other relevant explanatory variable which may affect the allocation decisions of the household sector [1].

Imposing the homogeneity condition:

$$y\_l1 + y\_l2 + y\_l3 + y\_l4 = 0 ,$$

equation (12) can be rewritten as:

$$(13) \quad \ln(Y_t) = y\_l0 + y\_lb \ln(SCALE_t) + y\_l2 \ln(PM_t / PCH_t) + y\_l3 \ln(USERIR_t / PCH_t) + y\_l4 \ln(PZ_t / PCH_t) + y\_l5\, H_t$$

for $Y_t = CPO_t, M_t / P_t, CIRO_t, Z_t,$ and with y = cp, m, cir, z .

Furthermore, using equations (11.a) to (11.c), we obtain:

$$(14) \quad \ln(Y_t) = y\_l0 + y\_lb \ln(SCALE_t) + y\_l2 \ln\left(\frac{LIC_t}{(1+LIC_t)}\right) + y\_l3 \ln\left(\frac{(1+LIC_t)PCIR_t - PCIR_{t+1}(1-\rho_{iro})}{(1+LIC_t)\ PCH_t}\right) - y\_l4 \ln\left(\frac{(1+LIC_t)\ PCH_t}{PCH_{t+1}}\right) + y\_l5\, H_t$$

for $Y_t = CPO_t, M_t / P_t, CIRO_t, Z_t,$ and with y = cp, m, cir, z .

1. For example, in the empirical section we include a dummy for German re-unification.

In other words, system (14) determines the demanded quantities of a particular good as a function of the available means, the nominal interest rate, the user cost of residential buildings, and the real interest rate. The theory of rational consumer behaviour indicates that the own price effects should be negative, i.e.:

$$cp\_l1, \quad m\_l2, \quad cir\_l3, \quad z\_l4 \quad < 0 .$$

2. Demand and the interest rate

In specification (14), there are three channels through which the interest rate affects demand.

First, there is the liquidity effect, measured by the term:

(15.a) $$y\_l2 \ln(LIC_t / (1+LIC_t)) \qquad \text{for } y = cp, m, cir, z .$$

When the nominal interest rate increases, the opportunity cost of money will increase and the demand for money will fall. The impact on the demand for the other goods and services is less clear a priori; it is an empirical issue to determine the exact sign of the elasticity. To know the semi-elasticity of the nominal interest rate one has to calculate [1]:

(15.b) $$y\_l2 \; / \; (LIC_t \, (1+LIC_t)) .$$

Second, there is the intertemporal substitution effect, measured by the term:

(15.c) $$- \, y\_l4 \ln((PCH_t \, (1+LIC_t)) / PCH_{t+1}) .$$

When the real interest rate increases, we expect that, ceteris paribus, the household sector will reduce its contemporaneous consumption and save more by holding interest-bearing assets. As a consequence we expect a negative relation between the real interest rate and contemporaneous consumption. The semi-elasticity of the interest rate through intertemporal substitution is found to be [2]:

(15.d) $$- \frac{y\_l4}{1 + LIC_t} .$$

Third, there is the impact of the interest rate on the user cost of residential buildings, measured by the term:

(15.e) $$y\_l3 \ln\left(\frac{PCIR_t}{PCH_t} - \frac{PCIR_{t+1}(1-\rho_{iro})}{PCH_t(1+LIC_t)}\right) .$$

1. Here use has been made of the fact that:
$dln(LIC / (1+LIC)) = dln(LIC) - dln(1+LIC) = d\,LIC \; / \; LIC - d\,(1+LIC) \; / \; (1+LIC) = 1 / (LIC\,(1+LIC)) \;\; d\,LIC.$
2. Here it should be remembered that $dln(1+LIC) = 1 / (1+LIC)\; d\,LIC$.

An increase in the nominal interest rate increases the user cost of residential buildings, and will decrease the demand for residential buildings. It is not a priori clear how the change in the user cost will affect the demand for consumption goods and the demand for money; they may be substitutes or complements.

The semi-elasticity of the interest rate through the user cost is found to be:

(15.f)
$$\frac{y\_l3}{\left(1+LIC_t-\frac{PCIR_{t+1}}{PCIR_t}(1-\rho_{iro})\right)} - \frac{y\_l3}{(1+LIC_t)}$$

where use has been made of equation (11.b) and the previous footnote.

Collecting terms, i.e. equations (15.b), (15.d) and (15.f), the overall semi-interest rate elasticity can be written as:

(15.g)
$$\frac{y\_l2}{(LIC_t(1+LIC_t))} - \frac{y\_l4}{1+LIC_t} + \frac{y\_l3}{\left(1+LIC_t-\frac{PCIR_{t+1}}{PCIR_t}(1-\rho_{iro})\right)} - \frac{y\_l3}{(1+LIC_t)} .$$

III The Empirical Results

In this section we show some empirical results for the EU, NE, US, and JP country blocks. First, we describe the data and we review briefly the main empirical regularities of the data. Next, we specify the short run dynamics and we discuss how we estimated the system under the assumption of rational expectations. Finally, some point estimates are given.

A. The data: sources and empirical regularities

We start by recalling the composition of the two aggregate country blocks, EU and NE. The ten EU block countries are Austria, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain. The four NE block countries are Denmark, Greece, Sweden, and the United Kingdom.

The data sources are described in Appendix B. Annual data are used and the sample size ranges from 1970 until 1996. The major data source is the National Accounts as published by the OECD, and, partly, available in the AMECO databank. Appendix C describes the trend and cyclical behaviour of some key variables of the household sector.

B. Towards empirical application

In the previous section we derived the long run demand equations of the household sector. However, here we assume that short run rigidities prevent immediate adjustment to this long run equilibrium. Datamining showed that the best fit was obtained if we assumed that the dynamics of private consumption and the demand for money are generated by an error correction mechanism, and that the dynamics of the expenditures on residential buildings are generated by a partial adjustment scheme.

1. An Error Correction Mechanism

Following the results of the above mentioned datamining, we postulate that an error correction mechanism captures the adjustment of private consumption and money demand towards its long run equilibrium. This error correction mechanism is of the form:

$$\Delta \ln Y_t = y\_sl(\ln Y_{t-1} - \ln \bar{Y}_{t-1}) + y\_sk \Delta \ln X_t + u_t \quad (16)$$

with $Y_t = CPO_t$, M_t / P_t, and y = cp, m, and with the variable u a stochastic variable. The variable X is short for any other variable that may affect the adjustment process. The superscript indicates the long run equilibrium value. Stability of the system requires that $-1 < y\_sl < 0$.

2. Partial adjustment [1]

The dynamics of the expenditures on residential buildings are best captured by a partial adjustment scheme.

Gross capital formation of residential buildings, $GIRO_t$, is defined as:

(17) $$GIRO_t = (CIRO_t - CIRO_{t-1}) + CIRO_{t-1}\,\rho_{iro}$$

with $CIRO_t$ defined as the stock of residential buildings at moment t, in constant prices.

We assume that there exist rigidities which prevent the contemporaneous stock of residential buildings, $CIRO_t$, from adjusting itself immediately to its desired level. This adjustment mechanism reads as follows:

(18) $$CIRO_t - CIRO_{t-1} = gir\_sl\,(CIROL_t - CIRO_{t-1})$$

with $CIROL_t$ defined as the desired stock of residential buildings at moment t, in constant prices.

For the parameter that measures the speed of adjustment, it holds that:

$0 < gir\_sl < 1$.

Inserting equation (18) into equation (17), yields:

(19.a) $$GIRO_t = gir\_sl\,(CIROL_t - CIRO_{t-1}) + CIRO_{t-1}\,\rho_{iro}\ .$$

Mutatis mutandis, this equation holds also for period t-1:

(19.b) $$GIRO_{t-1} = gir\_sl\,(CIROL_{t-1} - CIRO_{t-2}) + CIRO_{t-2}\,\rho_{iro}\ .$$

On subtracting $(1-\rho_{iro})$ times equation (19.b) from equation (19.a), we obtain:

(19.c) $$GIRO_t - (1-\rho_{iro})\,GIRO_{t-1} = gir\_sl\,(CIROL_t - (1-\rho_{iro})\,CIROL_{t-1}) - gir\_sl\,(CIRO_{t-1} - (1-\rho_{iro})\,CIRO_{t-2}) + \rho_{iro}\,(CIRO_{t-1} - (1-\rho_{iro})\,CIRO_{t-2})\ .$$

1. See, for example, Deaton and Muellbauer (1987, section 13.2) for a similar approach for durable consumption goods.

On rearranging terms and using the definition of $GIRO_t$, equation (19.c) is rewritten as:

$$(20) \qquad GIRO_t = gir\_sl \ (CIROL_t - (1-\rho_{iro})\ CIROL_{t-1}) + (1 - gir\_sl)\ GIRO_{t-1} .$$

Equation (20) explains contemporaneous gross fixed capital formation as a function of the change in the desired capital stock, and the lagged gross fixed capital formation.

Here we assume that the long run stock of residential buildings is described as [1]:

$$(21) \qquad CIROL_t = gir\_l0 + gir\_lb\ [DIH_t \ / \ PCH_t] + gir\_l1\ \ln[USERIR_t / PCH_t]$$

with: $gir\_l1 < 0$ and $gir\_lb > 0$.

Inserting equation (21) into equation (20), yields:

$$(22) \qquad GIRO_t = gir\_l0\ gir\_sl\ \rho_{iro} + gir\_sl\ \ gir\_lb\ \{ [DIH_t \ / \ PCH_t] - (1-\rho_{iro})\ [DIH_{t-1} \ / \ PCH_{t-1}] \} + gir\_sl\ \ gir\_l1 \{ \ln[USERIR_t / PCH_t] - (1-\rho_{iro})\ \ln[USERIR_{t-1} / PCH_{t-1}] \} + (1 - gir\_sl)\ GIRO_{t-1} .$$

We estimated equation (22) with ordinary least squares. The elasticities are defined as follows [2].

The short run income elasticity of gross fixed capital formation:

$$(23.a) \qquad gir\_sl\ gir\_lb\ \ mean\left(\frac{DIH}{PCH \ \ GIRO}\right) .$$

The long run income elasticity:

$$(23.b) \qquad gir\_lb\ \ \rho_{iro}\ \ mean\left(\frac{DIH}{PCH \ \ GIRO}\right) .$$

The short run elasticity of the user cost:

$$(23.c) \qquad gir\_sl\ \ gir\_l1\ mean\left(\frac{1}{GIRO}\right) .$$

The long run elasticity of the user cost:

$$(23.d) \qquad gir\_l1\ \rho_{iro}\ \ mean\left(\frac{1}{GIRO}\right) .$$

1. Implicitly we assume that the cross-elasticities of the other prices are equal to zero.
2. The long run elasticities are obtained by evaluating equation (22) for $GIRO_t = GIRO_{t-1} = GIRO$ and $DIH_t = DIH_{t-1} = DIH$.

3. Estimation

Before we show the estimates, the following remarks have to be made.

First, it should be noted that the behavioural equations include the expected value of the future consumer price index, PCH_{t+1}, and the expected value of the future price of residential buildings, $PCIR_{t+1}$. Since these variables cannot be observed directly, we need an additional assumption regarding the formation of expectations. Here, we assume rational expectations. In Appendix D it is described how this assumption has been implemented during estimation.

Second, in the empirical application all expenditures are defined as expenditures per capita, i.e., we divide the expenditures by total population, NPO.

Third, the interest rate LIC is a weighted average of the long run interest rate and the short run interest rate. The weight is equal to 0.5.

Fourth, the scale variable, SCALE, is measured by contemporaneous real disposable income. We impose a unit elasticity for the scale in the short and long run equation for money demand. No other restrictions are imposed during estimation.

Fifth, all equations are estimated with the Federal Planning Bureau's IODE software [1].

Sixth, we estimated the error correction mechanism using the Two-Step Engle-Granger method (see Engle and Granger (1991)) and we added some dummies to the equations. In the first stage we estimated the long run equilibrium equation:

$$(24)\ \ln\left(\frac{Y_t}{NPO_t}\right) = (y\_l0 + DUM7281\ y\_ld0) + y\_lb \ln\left(\frac{SCALE_t}{NPO_t}\right)$$

$$+ (y\_l2 + DUM7281\ y\_ld2) \ln\left(\frac{LIC_t}{(1+LIC_t)}\right)$$

$$+ (y\_l3 + DUM7281\ y\_ld3) \ln\left(\frac{(1+LIC_t)PCIR_t - \widehat{PCIR}_{t+1}(1-\rho_{iro})}{(1+LIC_t)\ \ PCH_t}\right)$$

$$- (y\_l4 + DUM7281\ y\_ld4) \ln\left(\frac{(1+LIC_t)\ \ PCH_t}{\widehat{PCH}_{t+1}}\right)$$

$$+ y\_l\_05\ \ DUMGE_t + y\_l\_06\ \ UKBUILD_t$$

for $Y_t = CPO_t$, M_t / P_t, y = cp, m, and with the $\frown$ symbol indicating a "rational expectations" value (see Appendix D).

---

1. See www.plan.be for more details regarding this software.

Several dummies were added to the original specification. DUMGE is a dummy to capture the effect of German re-unification, while UKBUILD is a dummy to capture the shift in the UK money data which was due to the inclusion of deposits of the building societies in the monetary aggregates as of 1987. The dummy DUM7281 is equal to one for the period ranging from 1972 until 1981, and equal to zero after 1981. The period until 1981 was a period of high inflation, and of significant inflation differences between countries of the EU and NE blocks. The dummies in equation (24) capture a structural break in aggregate household behaviour once the period of high inflation ended (see also Appendix C).

In the second step we estimated the short run adjustment mechanism:

$$(25)\quad \Delta\ln\left(\frac{Y_t}{NPO_t}\right) = y\_sb\ \Delta\ln\left(\frac{SCALE_t}{NPO_t}\right)$$

$$+\ y\_s2\ \Delta\ln\left(\frac{LIC_t}{1+LIC_t}\right) + y\_sd2\ \Delta\ \left(DUM7281\ \ln\left(\frac{LIC_t}{1+LIC_t}\right)\right)$$

$$+\ y\_s3\ \Delta\ln\left(\frac{(1+LIC_t)PCIR_t - \widehat{PCIR}_{t+1}(1-\rho_{iro})}{(1+LIC_t)\ \ PCH_t}\right)$$

$$+\ y\_sd3\ \Delta\ \left(DUM7281\ \ln\left(\frac{(1+LIC_t)PCIR_t - \widehat{PCIR}_{t+1}(1-\rho_{iro})}{(1+LIC_t)\ \ PCH_t}\right)\right)$$

$$-\ y\_s4\ \Delta\ln\left(\frac{(1+LIC_t)\ \ PCH_t}{\widehat{PCH}_{t+1}}\right) - y\_sd4\ \Delta\ \left(DUM7281\ \ln\left(\frac{(1+LIC_t)\ \ PCH_t}{\widehat{PCH}_{t+1}}\right)\right)$$

$$+\ y\_s5\ \Delta\ DUMGE_t + y\_s6\ \Delta\ UKBUILD_t$$

$$+\ y\_sl\ ECM_{t-1} + y\_s7\ \Delta\ln\left(\frac{Y_{t-1}}{NPO_{t-1}}\right)$$

for $Y_t = CPO_t$, M_t / P_t , y = cp, m, and where ECM_t is the error correction term derived from equation (24).

C. The empirical results

Tables 1 and 2 show estimates of the long run price and scale elasticities for private consumption and money demand for the EU, NE, US and JP blocks. Tables 3 and 4 show the estimates of the (semi-)elasticities for the error correction mechanism. Table 5 shows the estimates for gross fixed capital formation. Here we show only the most relevant estimation results. See Appendix E for a more detailed description of the estimation results.

TABLE 1 - Long Run (Semi-)Elasticities for Private Consumption

	EU	NE	US	JP
cp_l0 (constant)	-0.54	0.01	-0.45	-0.70
cp_lb (scale)	1.14	1.02	1.12	1.08
cp_l1 (opportunity cost M)	-0.03	0.09	-0.04	-0.01
cp_l3 (user cost res. bldg)	-0.01	-0.04	0.03	0.04
- cp_l4 (real interest rate)	-0.12	-0.56	-0.38	-0.77
Implicit interest semi-elasticity[a]	-0.60	-0.35	-0.46	-0.38
Diagnostic statistics				
R2-adjusted	1.00	0.99	1.00	0.99
Durbin Watson	2.16	1.70	1.92	0.95
Log Likelihood	90.73	79.67	86.10	64.40
Dickey Fuller	-5.11	-4.19	-4.84	-2.80
Augmented Dickey Fuller	-4.99	-4.10	-4.74	-2.73

a. See equation (15.g).

TABLE 2 - Long Run (Semi-)Elasticities for Money Demand

	EU	NE	US	JP
m_l0 (constant)	-0.90	-1.61	-1.66	-1.03
m_lb (scale)	1.00	1.00	1.00	1.00
m_l2 (opportunity cost M)	-0.01	-0.18	-0.04	-0.02
m_l3 (user cost res. bldg)	0.02	-0.02	-0.07	-0.09
- m_l4 (real interest rate)	-0.88	-0.57	-2.16	-1.74
Implicit interest semi-elasticity[a]	-0.43	-2.23	-3.50	-3.06
Diagnostic statistics				
R2-adjusted	0.97	0.96	0.72	0.96
Durbin Watson	1.56	1.45	0.69	1.23
Log Likelihood	68.81	39.28	42.35	52.71
Dickey Fuller	-3.76	-3.61	-2.63	-2.82
Augmented Dickey Fuller	-3.67	-3.53	-2.57	-2.72

a. See equation (15.g).

TABLE 3 - Short Run (Semi-)Elasticities for Private Consumption

	EU	NE	US	JP
cp_sb (scale)	0.93	0.94	1.01	1.08
	(0.11)	(0.12)	(0.07)	(0.12)
cp_s2 (opportunity cost M)	0.01	0.10	-0.02	0.04
	(0.03)	(0.04)	(0.02)	(0.02)
cp_s3 (user cost res. bldg)	-0.01	-0.02	0.02	-0.03
	(0.01)	(0.02)	(0.01)	(0.02)
- cp_s4 (real interest rate)	-0.40	-0.98	-0.45	-0.47
	(0.39)	(0.27)	(0.21)	(0.43)
cp_sl (ECM[-1])	-0.91	-0.95	-0.97	-0.52
	(0.26)	(0.31)	(0.22)	(0.18)
cp_s7 (lagged dependent)	0.22	0.09	0.00	0.00
	(0.10)	(0.14)	-.-	-.-
Implicit interest semi-elasticity[a]	-0.62	-0.28	-0.33	-0.24
Diagnostic statistics				
R2-adjusted	0.84	0.87	0.84	0.54
Durbin Watson	1.92	1.48	1.45	1.26
Log Likelihood	93.54	87.99	85.13	72.33

a. See equation (15.g).

TABLE 4 - Short Run (Semi-)Elasticities for Money Demand

	EU	NE	US	JP
m_sb (scale)	1.00	1.00	1.00	1.00
	-.-	-.-	-.-	-.-
m_s2 (opportunity cost M)	-0.06	-0.14	-0.16	-0.04
	(0.09)	(0.12)	(0.10)	(0.04)
m_s3 (user cost res. bldg)	0.03	0.01	0.03	-0.06
	(0.02)	(0.02)	(0.06)	(0.03)
- m_s4 (real interest rate)	-0.38	-0.43	-1.38	-1.22
	(1.13)	(0.69)	(0.98)	(0.42)
m_sl (ECM[-1])	-0.80	-0.46	-0.36	-0.62
	(0.31)	(0.26)	(0.17)	(0.24)
Implicit interest semi-elasticity[a]	-0.08	-1.35	-2.77	-2.49
Diagnostic statistics				
R2-adjusted	0.80	0.50	0.62	0.74
Durbin Watson	1.66	1.44	0.86	1.58
Log Likelihood	67.96	42.36	51.08	55.03

a. See equation (15.g).

TABLE 5 - Elasticities for Gross Fixed Capital Formation Residential Buildings

	EU	NE	US	JP
Short Run elasticities[a]				
Scale	1.09	2.20	4.77	2.86
User cost of res. building	-0.26	-0.69	-0.42	-0.90
Long Run elasticities[b]				
Scale	0.33	0.27	0.27	0.32
User cost of res. building	-0.08	-0.08	-0.02	-0.10
Partial adjustment coef.	0.03	0.19	0.47	0.29
Diagnostic statistics				
R2-adjusted	0.75	0.70	0.55	0.73
Durbin Watson	1.34	1.03	1.85	1.77
Log Likelihood	60.69	9.11	27.36	-90.61

a. See equations (23.a) and (23.c).

b. See equations (23.b) and (23.d).

The tables show also some diagnostic statistics. First, there is the traditional adjusted R-squared and the Durbin Watson (see Johnston (1984)), while the Dickey Fuller (DF) statistic refers to the order of integration of the error term of the long run equations (see Charemza and Deadman, (1993)) [1].

Let us now have a look at the results. First, we start with the error correction mechanisms for private consumption and money demand. The diagnostic statistics are fairly good. In most cases the adjusted R-squared is high indicating that a fair amount of variation in the data has been explained. The Durbin Watson statistics are also fairly good. The DF statistics indicate that we can reject the null-hypothesis of no-cointegration at a fair level of confidence.

We obtain that for private consumption the long run scale elasticity is larger than one for all blocks, and larger than the short run scale elasticities.

As indicated earlier, all own price effects should be negative. The a priori sign of the cross-price elasticities is less evident. Comparing the results across the different blocks, we see that most parameters have the same sign. For example, the elasticity of the real interest rate is negative in all private consumption and money demand equations. The elasticity of the opportunity cost of holding money, i.e. the nominal interest rate, is negative in all the money demand equations. Finally, note that the sign of the elasticity of the user cost of residential buildings in private consumption and in the money demand function differs across countries.

1. Here, the null-hypothesis of no cointegration is tested against the alternative hypothesis of cointegration. The area of rejection of the null-hypothesis is the area for which the DF test statistic without intercept is smaller than -1.99 (the test is indecisive for values between -1.99 and -1.84) at the 5 percent confidence level, and the area for which the DF test statistic with intercept is smaller than -2.33 (the test is indecisive for values between -2.33 and -2.11) at the 5 percent confidence level.

The row “Implicit interest semi-elasticity” measures the total impact of a change in the interest rate, as defined in equation (15.g). Indeed, recall that the interest rate affects demand through three channels: the liquidity effect, the intertemporal substitution effect, and the user cost effect. The numbers presented in this row summarize the total impact of a 100 points increase in the interest rate. We see that an interest rate increase, decreases the demand for goods, money and residential buildings in the short run as well as in the long run. These results indicate, for example, that if the interest rate increases by 100 base points, private consumption in the EU block will decrease by 0.6 percent, ceteris paribus. Likewise, we see a 0.5 percent drop in US private consumption, when the US interest rate increases 100 base points. Not surprisingly, the demand for money generally has the largest overall semi-elasticity.

All error correction terms are between 0 and -1, indicating convergences to a steady state. A low value indicates that adjustment to a shock occurs slowly. The partial adjustment coefficient of the demand for residential buildings indicates a slow adjustment process.

The point estimates for gross fixed capital formation of residential buildings show that in the short run there are considerable differences in the elasticities. The high income elasticity of the US reflects the finding (see Appendix C) that the gross fixed capital formation series is a rather volatile one. However, in the long run the scale elasticities seem to be similar across blocks.

IV Conclusion

This paper presented some results for the household sector of the NIME model.

In the first section, we derived the long run expenditure plans of the household sector, and we showed how private consumption goods, the demand for money, and household gross fixed capital formation are determined by the available means, the nominal interest rate, the real interest rate, and the opportunity cost of residential buildings.

In the empirical section of the paper we made the additional assumptions that rigidities prevent the household sector from adjusting its expenditures immediately to its long run equilibrium plan, and that the adjustment process to the long run equilibrium can be captured by an error correction model and a partial adjustment process. We presented results for private consumption, money demand, and investment in residential buildings. More precisely, we showed estimates for the long and short run responses of private consumption, money demand, and investment, to changes in income, the nominal and real interest rates, and the user cost of residential buildings. There we obtained, for example, that if the interest rate increases by 100 base points, private consumption in the EU block will decrease by 0.6 percent, ceteris paribus. Similar results were obtained for the other blocks.

In future papers we will describe the rest of the NIME model. There is also the possibility of future improvement in the fit of the household equations, for example, by using a more refined scale concept, and by deriving explicitly the short run adjustment schemes.

V

Appendix A: The Optimization Problem of the Household Sector

The household sector maximizes its intertemporal utility function, equation (8), subject to its intertemporal budget constraint, equation (3.c), i.e.:

$$\text{(A.1)} \qquad \underset{CPO_t,\, M_t,\, CIRO_t,\, CAOU_t,\, INVHO_t}{MAX} \; U1\,(CPO_t,\, M_t / PCH_t,\, CIRO_t,\, Z_t)$$

subject to:

$$\begin{aligned} & CIRO_{t-1}\,(1-\rho_{iro})\; PCIR_t + INVHO_{t-1}\,(1-\rho_{invh})\; PINVH_t \\ & + M_{t-1} + CAOU_{t-1}\,(1{+}LIC_{t-1}) + WRP_t\, NP_t + WRG_t\, NG_t + UB_t\,(LS_t - (NP_t + NG_t)) \\ & + TRANS\_00_t + NOIH_t - DTH_t - DTC_t \\ & = CPO_t\, PCH_t + M_t + CIRO_t\, PCIR_t + INVHO_t\, PINVH_t + CAOU_t \end{aligned}$$

with

$$\begin{aligned} Z_t = {} & (M_t + CIRO_t (1-\rho_{iro})\, PCIR_{t+1} + INVHO_t\,(1-\rho_{invh})\, PINVH_{t+1} \\ & + CAOU_t\,(1{+}LIC_t) + ZY_{t+1}) / PCH_{t+1}\,. \end{aligned}$$

See sections II.A.1 and II.A.2 of the main text for more details.

The Lagrangian function of this problem reads as:

$$\begin{aligned} \text{(A.2)} \qquad L = {} & U1\,(CPO_t,\, M_t / PCH_t,\, CIRO_t,\, Z_t) \\ & - \lambda\;(\,(M_t - M_{t-1}) + (CIRO_t - CIRO_{t-1}\,(1-\rho_{iro})\,)\, PCIR_t \\ & + (INVHO_t - INVHO_{t-1}\,(1-\rho_{invh})\,)\, PINVH_t + (CAOU_t - CAOU_{t-1}) \\ & - (WRP_t\, NP_t + WRG_t\, NG_t + UB_t\,(LS_t - (NP_t + NG_t)) \\ & + (TRANS\_00_t + NOIH_t - DTH_t - DTC_t) + CAOU_{t-1}\, LIC_{t-1} - CPO_t\, PCH_t)). \end{aligned}$$

The first order conditions for an optimum of (A.2) are [1]:

(A.3.a) $L_{CPO_t} = U1_{CPO_t} - \lambda\ PCH_t = 0$

(A.3.b) $L_{M_t} = U1_{(M_t/PCH_t)} \frac{1}{PCH_t} + U1_{Z_t} \frac{1}{PCH_{t+1}} - \lambda = 0$

(A.3.c) $L_{CIRO_t} = U1_{CIRO_t} + U1_{Z_t} \frac{(1-\rho_{iro})PCIR_{t+1}}{PCH_{t+1}} - \lambda\ PCIR_t = 0$

(A.3.d) $L_{CAOU_t} = U1_{Z_t} \frac{1+LIC_t}{PCH_{t+1}} - \lambda = 0$

(A.3.e) $L_{INVH_t} = U1_{Z_t}\ (1-\rho_{invh}) \frac{PINVH_{t+1}}{PCH_{t+1}} - \lambda\ PINVH_t = 0$

(A.3.f) $(M_t - M_{t-1}) + (CIRO_t - CIRO_{t-1}\ (1-\rho_{iro}))\ PCIR_t$
$+ (INVHO_t - INVHO_{t-1}\ (1-\rho_{invh}))\ PINVH_t + (CAOU_t - CAOU_{t-1})$
$- (WRP_t\ NP_t + WRG_t\ NG_t + UB_t\ (LS_t - (NP_t + NG_t))$
$+ (TRANS\_00_t + NOIH_t - DTH_t - DTC_t) + CAOU_{t-1}\ LIC_{t-1} - CPO_t\ PCH_t) = 0$.

Using condition (A.3.d), we can rewrite conditions (A.3.b) and (A.3.c) as:

(A.4.a) $L_{M_t} = U_{(M_t/PCH_t)} - \lambda \frac{LIC_t}{1+LIC_t} PCH_t = 0$

(A.4.b) $L_{CIRO_t} = U_{CIRO_t} - \lambda\ (PCIR_t - \frac{PCIR_{t+1}(1-\rho_{iro})}{1+LIC_t}) = 0$

or

(A.5.a) $U_{(M_t/P_t)} = \lambda \frac{LIC_t}{1+LIC_t} PCH_t$

(A.5.b) $U_{CIRO_t} = \lambda\ (PCIR_t - \frac{PCIR_{t+1}(1-\rho_{iro})}{1+LIC_t})$.

Note also that from (A.3.d) and (A.3.e) it follows that:

(A.6) $$\frac{PINVH_{t+1}}{PINVH_t} = \frac{1+LIC_t}{1-\rho_{invh}}$$

i.e. equation (A.6) defines a restriction on the path of the price of inventories.

1. Notation: $U1_x \equiv \frac{\partial}{\partial x} U1(......)$.

We now proceed to define the vectors Y_t and Π_t:

$$Y_t' = (CPO_t, \frac{M_t}{PCH_t}, CIRO_t, Z_t)$$

and

$$\Pi'_t = (PCH_t, \frac{LIC_t}{(1+LIC_t)} PCH_t, PCIR_t - \frac{PCIR_{t+1}(1-\rho_{iro})}{1+LIC_t}, \frac{PCH_{t+1}}{1+LIC_t})$$

so that the previous marginal equilibrium conditions can be written as:

(A.7) $$U_Y = \lambda \ \Pi_t \ .$$

In other words, the marginal utility of the goods Y is proportional to the prices Π. Note also that:

(A.8)
$$\begin{aligned}
\Pi'_t Y_t &= PCH_t\, CPO_t + PCH_t\, LIC_t \,/\,(1+LIC_t)\, M_t \,/\, PCH_t \\
&+ (PCIR_t - (1-\rho_{iro})\, PCIR_{t+1} \,/\, (1+LIC_t))\, CIRO_t \\
&+ PCH_{t+1} \,/\, (1+LIC_t)\, (\, M_t + CIRO_t\, (1-\rho_{iro})\ PCIR_{t+1} \\
&+ INVHO_t\, (1-\rho_{invh})\ PINVH_{t+1} \\
&+ CAOU_t\, (1+LIC_t) + ZY_{t+1}\,) \,/\, PCH_{t+1} \\
&= PCH_t\, CPO_t + M_t + PCIR_t\, CIRO_t + CAOU_t + ZY_{t+1} \,/\, (1+LIC_t) \\
&= EXHU_t + ZY_{t+1} \,/\, (1+LIC_t) = WHU_t + ZY_{t+1} \,/\, (1+LIC_t) \quad \equiv \quad PERM_t
\end{aligned}$$

with PERM the permanent income, i.e. assets inherited from the past, plus contemporaneous income, plus discounted future non-asset income. Note that use has been made of condition (A.6).

VI Appendix B: The Data

A. Household expenditure and revenue

We use annual data, ranging from 1970 until 1996. The main source of the data on household expenditure and income is the AMECO databank, which uses the National Accounts, as published, for example, by the OECD (Table 8. Accounts for Households) and EUROSTAT.

B. Financial data

M: Money is M1, i.e., line 34, Money, of International Financial Statistics, of International Monetary Fund.

LIC: a weighted average of the nominal long run interest rate, AMECO series XXO1ILN, and the nominal short run interest rate, AMECO series XXO1ISN. The weight is equal to 0.5, i.e. XX_LIC = 0.5 XXO1ILN + 0.5 XXO1ISN.

C. Missing observations

Missing observations have been interpolated. See Barten (1984).

D. The definition of the aggregates of the country blocks

The two aggregate country blocks, EU and NE, are composed as follows. The ten EU block countries are Austria, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain. The four NE block countries are Denmark, Greece, Sweden, and the United Kingdom.

For notational convenience we introduce here the label XX, with XX = EU, NE, US, JP.

1. The unit of account and exchange rates

For the blocks consisting of more than one country, a new currency unit has been defined. For the EU block this currency unit is the euro [1]. For the NE block the unit is a weighted average of the currencies of Denmark, Greece, Sweden, and the United Kingdom.

2. Aggregation of the expenditures

For block XX we define the expenditures in current prices as:

$$(B.1) \qquad XX\_ZU = \sum_{\forall i \in XX} i\_ZU \quad PPP\_XX\_i$$

where:

XX_ZU: expenditures in current prices for Z in block XX, denominated in the currency of block XX,

i_ZU: expenditures in current prices for Z in country i, denominated in local currency,

PPP_XX_i: the purchasing power parity exchange rate, number of units of the currency of block XX, per unit of the currency of country i.

We define the expenditures in constant prices as:

$$(B.2) \qquad XX\_ZO = \sum_{\forall i \in XX} i\_ZO \quad PPP\_XX\_i_{1990}$$

where:

XX_ZO: expenditures in constant prices for Z in block XX, denominated in the currency of block XX,

i_ZO: expenditures in constant prices for Z in country i, denominated in the local currency,

$PPP\_XX\_i_{1990}$: the purchasing power parity exchange rate, number of units of the currency of block XX, per unit of the currency of country i.

3. Prices

The corresponding prices are defined as:

$$(B.3) \qquad XX\_PZ = \frac{XX\_ZU}{XX\_ZO} .$$

1. The ECU before 1999.

4. The aggregate interest rate

For each block XX we define the short and long run interest rates as:

$$\text{(B.4.a)} \qquad XX\_SI = \sum_{\forall i \in XX} i\_SI \ \ \omega_i$$

and

$$\text{(B.4.b)} \qquad XX\_LI = \sum_{\forall i \in XX} i\_LI \ \ \omega_i$$

with the weights, ω_i, defined as:

$$\omega_i = \text{i\_GDPU} / \left(\sum_{\forall j \in XX} \text{j\_GDPU} \right) \qquad \forall i \in XX$$

with i_GDPU: gross domestic product in current prices.

VII Appendix C: Empirical Regularities of Some Key Variables

This appendix describes some empirical regularities of the household sector of the NIME model. First we look at the trend behaviour of some key variables, next we show how variables deviate *together* from their trend, and we also describe the persistence of the deviations from trend.

Table C1 highlights the importance of the household sector in the economy by showing the (average) share in Gross Domestic Product (GDP) of private consumption and of gross fixed capital formation of residential buildings. The first column lists the name of the variable for which the ratio is calculated. The second column shows the average ratio for the period ranging from 1971 until 1996, while the third column presents the standard deviation of this ratio. The next three columns show the average ratio for the periods 1971-1980, 1981-1990, and 1991-1996.

The evidence in table C1 indicates that in all blocks the share of private consumption increased slightly during the course of time, while the share of gross fixed capital formation in residential buildings declined. The share of private consumption in GDP is, on average, at about 60 percent in the EU, JP and NE blocks, and about 65 percent in the US. In the US the share of private consumption rose more than in the other blocks, i.e., from 63 percent in 1971-1980 to 68 percent in 1991-1996.

Table C1 shows also that real per capita GDP and real per capita household disposable income, measured in PPP-euro terms, was highest in the US throughout the sample period, and that GDP per capita in Japan was lowest in 1971-1980, but second highest in 1991-1996.

TABLE C1 - Average share of GDP of key variables

	1971-1996	Standard deviation	1971-1980	1981-1990	1991-1996
Private consumption					
EU block	0.594	0.008	0.588	0.596	0.598
NE block	0.607	0.013	0.599	0.605	0.625
US block	0.650	0.021	0.630	0.653	0.679
JP block	0.579	0.019	0.563	0.588	0.589
Gross fixed capital formation of residential buildings					
EU block	0.059	0.009	0.069	0.055	0.049
NE block	0.040	0.006	0.046	0.039	0.032
US block	0.046	0.007	0.051	0.044	0.039
JP block	0.063	0.011	0.075	0.056	0.054
Pro memori					
GDP per capita in 1990 prices (thousands of PPP-euro)					
EU block	12.647	1.855	10.843	13.160	14.799
NE block	12.268	1.701	10.645	12.704	14.246
US block	18.481	2.195	16.441	18.936	21.122
JP block	13.332	3.056	10.337	13.894	17.388
Disposable household income per capita in 1990 prices (thousands of PPP-euro)					
EU block	8.594	1.152	7.525	8.823	9.995
NE block	7.804	1.229	6.670	7.937	9.470
US block	13.311	1.722	11.608	13.806	15.324
JP block	9.398	1.773	7.787	9.593	11.756
Share of household savings in disposable income					
EU block	0.140	0.025	0.166	0.128	0.116
NE block	0.068	0.022	0.081	0.050	0.078
US block	0.091	0.017	0.103	0.090	0.070
JP block	0.168	0.035	0.205	0.150	0.135

A. Trend Behaviour

Tables C2 and C3 describe the trend behaviour of some expenditure items and prices, respectively. The structure of these tables is as follows. The first column shows the name of the variable. The second column shows the average growth rate of the variable over the whole sample period. The third column presents the standard deviation of the growth rate. The next three columns show the growth rates for the periods 1971-1980, 1981-1990, and 1991-1996. The sixth column shows the autocorrelation coefficient RHO, while the seventh and eight column present the Dickey Fuller and Augmented Dickey Fuller test statistic, respectively [1].

The evidence in table C2 indicates that the average growth rate of private consumption was larger than the average growth rate of GDP in all country blocks. Except for the NE block, the growth of private consumption was highest in 1971-1980, and lowest in 1991-1996. In the NE block the highest growth rate was recorded in 1981-1990.

The average growth rate of gross fixed capital formation in residential buildings was highest in the US, and lowest in the NE block. This growth rate was even negative in the NE and JP blocks during the period 1991-1996. Of special interest is to note the rather high standard deviation of the growth of gross capital formation in the US. The growth rates of real money balances differ quite strongly across blocks: declining in the EU block, starting from a negative average value in the NE and US blocks, increasing during the eighties and then slowing down in the NE block in the nineties.

Table C3 provides some evidence on prices. The highest average price increases are recorded in the NE block, and the lowest in Japan. Clearly, during the 1971-1980 period there was a much higher inflation rate than during the 1991-1996 period. Note that the NE block had the highest average increase and the highest standard deviation for the GDP deflator, i.e., 8.3 percent and 5.1 percent, respectively. The interest rates were lowest during the 1990-1996 period.

---

1. For convenience, we summarize here briefly some general notions on unit roots. For a thorough introduction to unit roots, see Maddala and Kim (1999), or Charemza and Deadman (1993). The Dickey Fuller test is defined as follows. The starting point is equation (C.1):
   (C.1) $X_t = RHO\ X_{t-1} + u_t$.
   Subtracting X_{t-1} from both sides of equation (C.1), we obtain:
   (C.2) $d\ X_t = B\ X_{t-1} + u_t$ with $B = RHO - 1$.
   Under the assumption that u_t is white noise, one proceeds by estimating B in equation (C.2), and testing $H_0 : B = 0$, i.e. RHO = 1 or a unit root; i.e., X integrated of order 1; against $H_1 : B < 0$, i.e., X integrated of order zero. Lower and upper critical values are provided for the Dickey Fuller (DF) test statistics in, for example, Charemza and Deadman (1993). If the computed Student t-statistic is smaller than the lower critical value for a particular number of observations then the null hypothesis is rejected. If the t-statistic is greater than the upper critical value then the null hypothesis cannot be rejected. If the t-statistic is between the upper and lower critical values then the test is indecisive. In our exercise the lower and upper critical values for the DF statistic are, at the 1 percent level of significance -2.80, and -2.48, respectively, and, at the 5 percent level of significance, -1.99 and -1.84, respectively.
   The Dickey Fuller test is based on the assumption that ut is white noise. When this assumption is not met, one calculates the Augmented Dickey Fuller (ADF) test statistic, by estimating
   $$(C.3)\ d\ X_t = B\ X_{t-1} + \sum_{k=1,n} g_k\ d\ X_{t-k} + u_t$$
   with n chosen in such a way that u_t is white noise. The ADF test statistic reported in tables 2 until 5 also includes a constant (drift). For the ADF statistic with k = 3 and with a drift, the lower and upper critical values at the 1 percent level of significance, are -4.88 and -4.53, respectively, and at the 5 percent level of significance, -3.96 and -3.82, respectively.

TABLE C2 - Trend behaviour of key variables in constant prices

	Average growth rate					RHO	Dickey Fuller	Augmented
	1971-1996	Standard deviation	1971-1980	1981-1990	1991-1996			Dickey Fuller
Private consumption								
EU block	2.828	1.696	3.652	2.427	2.188	1.026	7.475	0.117
NE block	2.381	2.126	2.320	3.052	1.020	1.023	6.045	0.934
US block	2.949	1.683	3.115	3.100	2.315	1.028	9.971	1.677
JP block	3.740	2.172	4.734	3.724	1.949	1.033	9.098	0.490
Gross fixed capital formation for residential buildings								
EU block	0.526	3.309	0.822	0.226	0.528	1.004	0.653	-1.514
NE block	-0.182	7.326	0.018	1.452	-3.854	0.993	-0.263	-2.553
US block	3.270	15.240	3.401	2.782	3.460	1.012	0.409	-1.188
JP block	2.734	8.121	2.969	3.658	-1.656	1.020	0.873	-0.914
Real M1								
EU block	3.252	5.762	4.958	2.525	1.100	1.026	3.006	0.408
NE block	3.471	6.319	-0.189	7.184	2.768	1.040	3.906	1.571
US block	1.319	4.870	-0.151	2.211	2.865	1.012	1.302	0.291
JP block	4.404	6.692	3.812	3.511	6.340	1.046	4.502	4.112
Disposable income								
EU block	2.499	1.976	3.215	2.175	2.008	1.023	5.692	-0.287
NE block	2.344	2.299	2.405	2.453	1.917	1.023	5.834	1.210
US block	2.768	1.703	3.140	2.739	2.151	1.026	8.765	1.331
JP block	3.579	2.405	4.814	3.029	2.315	1.031	7.742	0.356
Pro memori								
GDP								
EU block	2.621	1.655	3.254	2.416	1.954	1.024	7.760	0.527
NE block	2.118	1.931	2.202	2.431	1.290	1.020	5.926	1.149
US block	2.643	2.088	2.839	2.625	2.101	1.026	7.297	1.778
JP block	3.698	2.191	4.495	4.004	1.449	1.032	7.879	0.074
Household saving								
EU block	0.669	6.524	1.120	0.739	0.743	1.002	0.192	-2.109
NE block	5.827	30.343	4.661	0.591	19.908	1.007	-0.558	-3.010
US block	1.302	11.329	3.812	-0.235	0.655	0.999	-0.417	-1.339
JP block	2.939	7.771	5.553	-0.724	4.840	1.019	0.902	-3.231

TABLE C3 - Trend behaviour of prices of key variables

	Average growth rate					RHO	Dickey Fuller	Augmented
	1971-1996	Standard deviation	1971-1980	1981-1990	1991-1996			Dickey Fuller
Price level of private consumption								
EU block	6.297	3.605	9.338	5.113	3.138	1.044	7.979	-1.025
NE block	8.157	4.806	12.623	6.265	4.138	1.052	8.446	-0.916
US block	5.219	2.422	7.145	4.790	2.870	1.041	9.856	-1.281
JP block	4.446	4.784	8.924	2.052	1.144	1.027	4.254	-5.823
Price level of gross fixed capital formation for residential buildings								
EU block	7.490	5.953	13.277	5.091	1.859	1.041	5.176	-2.022
NE block	9.341	6.899	16.395	5.011	5.665	1.056	6.855	0.266
US block	5.669	3.342	9.138	4.055	2.582	1.040	8.466	-0.507
JP block	4.562	6.349	9.624	1.458	1.482	1.028	3.522	-2.485
Nominal short run interest rate (percent)								
EU block	9.550	2.433	9.231	10.788	8.242	0.974	-0.690	-1.993
NE block	10.480	2.651	10.219	12.106	8.614	0.981	-0.548	-1.226
US block	6.974	2.552	6.871	8.508	4.705	0.970	-0.640	-1.701
JP block	6.173	2.932	8.006	5.955	3.128	0.914	-1.319	-0.701
Nominal long run interest rate (percent)								
EU block	10.182	1.903	10.044	11.306	8.781	0.990	-0.463	-1.556
NE block	11.559	1.931	12.219	12.105	9.960	0.994	-0.401	-0.870
US block	8.251	2.077	7.293	10.152	6.958	0.992	-0.350	-1.881
JP block	6.527	1.703	7.685	6.562	4.430	0.974	-1.093	0.182
Pro memori								
Implicit GDP deflator								
EU block	6.370	3.304	9.448	5.299	3.012	1.044	8.675	-1.116
NE block	8.318	5.110	13.096	6.429	3.654	1.051	8.111	-0.985
US block	5.234	2.567	7.361	4.758	2.693	1.039	8.872	-1.575
JP block	3.936	4.593	7.894	1.930	0.910	1.024	3.884	-5.844

B. Cyclical behaviour

Tables C4 and C5 describe the cyclical behaviour of key variables. Here, we measure the cyclical component of a variable as the difference between the original series and its trend. The latter is obtained using the Hodrick-Prescott (HP) filter [1].

The structure of tables C4 and C5 is as follows. The first column lists the name of the series for which the cyclical component is examined. The second column shows the standard error of the cyclical component of the variable divided by the standard error of the cyclical component of GDP [2]. The third column shows the autocorrelation coefficient [3] that measures the persistence in the deviation from the trend. The sixth column shows the correlation between the contemporaneous cyclical component of GDP and the contemporaneous cyclical component of the other series. Columns five and four show the correlation between contemporaneous GDP and the variable lagged one and two periods, respectively, while columns seven and eight show the results for one and two leads, respectively. These coefficients illustrate co-movements, and not necessarily a causal link.

Let us now have a closer look at each of these tables. Private consumption is quite stable. It deviates less from its trend than GDP (see column 2), except for the EU and NE blocks. Gross fixed capital formation in residential buildings is much more volatile than GDP (see column 2). Money (M1) also fluctuates more around its trend than GDP. Almost all components of aggregate demand are procyclical. Finally, note that the variability of the short term interest rates is much higher than the variability of the long term rates.

1. i.e., IODE-procedure: $wstrend with the smoothing parameter lambda equal to 100. For a thorough discussion of these filters see Canova (1998a) and (1998b), and Burnside (1998).
2. i.e., stderr(1971Y1, 1996Y1, ln(X / HPX)) / stderr(1971Y1, 1996Y1, ln(GDP / HPGDP)), with X the original series and HPX the HP-trend of variable X.
3. For a variable X the autocorrelation coefficient, RHO, is obtained estimating ln(Xt / HPXt) = RHO ln(Xt-1 / HPXt-1) + ut, with u a random component.

TABLE C4 - Cyclical behaviour of key variables

	SD[a]	RHO[b]	Cross-correlation of GDP with X[c]				
			X[t-2]	X[t-1]	X[t]	X[t+1]	X[t+2]
Private consumption							
EU block	1.149	0.745	0.113	0.569	0.908	0.712	0.346
NE block	1.110	0.693	-0.036	0.571	0.943	0.784	0.393
US block	0.859	0.586	0.124	0.728	0.924	0.338	-0.247
JP block	0.841	0.465	-0.092	0.490	0.906	0.519	0.117
Gross fixed capital formation for residential buildings							
EU block	1.669	0.569	-0.110	0.419	0.773	0.666	0.316
NE block	2.281	0.586	0.219	0.744	0.889	0.501	0.043
US block	7.360	0.499	0.296	0.803	0.781	0.006	-0.515
JP block	3.218	0.663	0.432	0.671	0.540	0.005	-0.240
Real M1							
EU block	2.440	0.677	0.432	0.857	0.715	0.381	-0.006
NE block	3.134	0.769	0.227	0.742	0.852	0.684	0.348
US block	2.366	0.576	0.295	0.532	0.456	0.111	-0.272
JP block	1.867	0.223	0.011	0.438	0.453	-0.170	-0.413
Pro memori							
Household savings							
EU block	3.593	0.514	-0.287	0.262	0.602	0.721	0.576
NE block	14.523	0.666	-0.559	-0.734	-0.642	-0.336	0.033
US block	4.200	0.334	-0.479	-0.737	-0.343	-0.028	0.189
JP block	3.225	0.683	-0.407	-0.510	-0.235	0.102	0.241
Nominal M1							
EU block	1.712	0.672	0.090	0.664	0.719	0.573	0.274
NE block	2.448	0.749	0.109	0.595	0.683	0.668	0.500
US block	1.850	0.534	0.203	0.325	0.263	0.054	-0.217
JP block	1.816	0.603	-0.178	0.030	0.096	-0.125	-0.179
GDP							
EU block	1.740	0.621	0.074	0.584	1.000	0.631	0.188
NE block	2.317	0.672	0.082	0.653	1.000	0.713	0.181
US block	2.149	0.504	-0.157	0.515	1.000	0.528	-0.104
JP block	2.030	0.546	-0.070	0.566	1.000	0.617	0.133

a. For all variables except GDP, SD is defined as:
SD = sterr(1971Y1,1996Y1,ln(X/HPX)) / sterr(1971Y1,1996Y1,ln(GDP/HPGDP)), with X the original series, and HPX the HP-trend of X. For GDP, SD is defined as SD = sterr(1971Y1,1996Y1,ln(GDP/HPGDP))

b. For a variable X the autocorrelation coefficient, RHO, is obtained estimating ln(X/HPX)t = RHO ln(X/HPX)t-1 + ut, with u a random component.

c. i.e., corr(ln(X/HPX)[t+y], ln(GDP/HPGDP)[t]) with y = -2, -1, 0, 1, 2

TABLE C5 - Cyclical behaviour of prices of key variables

	SD[a]	RHO[b]	Cross-correlation of GDP with X[c]				
			X[t-2]	X[t-1]	X[t]	X[t+1]	X[t+2]
Price level of private consumption							
EU block	1.302	0.663	-0.604	-0.620	-0.343	0.041	0.345
NE block	0.920	0.660	-0.300	-0.618	-0.691	-0.401	0.129
US block	0.995	0.746	-0.344	-0.714	-0.641	-0.179	0.264
JP block	1.104	0.532	-0.246	-0.448	-0.462	0.035	0.284
Price level of gross fixed capital formation for residential buildings							
EU block	1.810	0.595	-0.251	-0.102	0.220	0.564	0.609
NE block	1.801	0.786	-0.628	-0.587	-0.329	0.051	0.239
US block	1.377	0.812	-0.522	-0.448	-0.103	0.283	0.386
JP block	1.422	0.612	-0.444	-0.478	0.029	0.576	0.555
Consumer price inflation							
EU block	2.110	0.490	-0.295	-0.091	0.333	0.474	0.385
NE block	2.733	0.433	-0.335	-0.377	-0.073	0.513	0.703
US block	1.423	0.519	-0.619	-0.500	0.116	0.705	0.724
JP block	4.972	0.818	-0.221	-0.190	0.029	0.321	0.184
Nominal short term interest rate							
EU block	1.827	0.529	-0.450	-0.205	0.477	0.644	0.525
NE block	1.881	0.515	-0.693	-0.487	0.213	0.714	0.732
US block	1.837	0.595	-0.817	-0.576	0.170	0.483	0.455
JP block	2.180	0.407	-0.260	-0.099	0.306	0.768	0.549
Nominal long term interest rate							
EU block	1.235	0.636	-0.434	-0.202	0.235	0.463	0.474
NE block	1.094	0.434	-0.577	-0.589	-0.145	0.477	0.768
US block	1.086	0.524	-0.574	-0.655	-0.316	-0.022	0.207
JP block	0.946	0.510	-0.305	0.029	0.338	0.558	0.478
Pro memori							
Implicit GDP deflator							
EU block	1.999	0.651	-0.560	-0.506	-0.208	0.178	0.434
NE block	3.638	0.663	-0.397	-0.604	-0.595	-0.272	0.279
US block	1.998	0.780	-0.281	-0.664	-0.640	-0.290	0.145
JP block	2.668	0.502	-0.254	-0.435	-0.351	0.231	0.316

a. For all variables except GDP, SD is defined as:
SD = sterr(1971Y1,1996Y1,ln(X/HPX)) / sterr(1971Y1,1996Y1,ln(GDP/HPGDP)) with X the original series, and HPX the HP-trend of X. For GDP, SD is defined as SD = sterr(1971Y1,1996Y1,ln(GDP/HPGDP))

b. For a variable X the autocorrelation coefficient, RHO, is obtained estimating ln(X/HPX)t = RHO ln(X/HPX)t-1 + ut, with u a random component.

c. i.e., corr(ln(X/HPX)[t+y], ln(GDP/HPGDP)[t]) with y = -2, -1, 0, 1, 2

VIII Appendix D: Estimation under the Assumption of Rational Expectations

In this appendix we discuss the introduction of the assumption of rational expectations during estimation; see Cuthberston et al. (1992) for more details on the estimation under the assumption of rational expectations.

A. An outline of the problem

Consider the following simple model:

(D.1) $$Y_t = a + b\ X_t + c\ E_t(X_{t+1}) + u_t$$

with $E_t(.)$ the expectations operator, conditional on all relevant information. Y_t and X_t are non-stochastic variables, and u_t is a stochastic variable with zero mean and constant variance.

In other words, $E_t(X_{t+1})$ is the expected value at *t* of the variable X in period *t+1*. However, $E_t(X_{t+1})$ cannot be observed. To circumvent this problem we proceed as follows. Under rational expectations we have that:

(D.2) $$X_{t+1} = E_t(X_{t+1}) + v_{t+1}$$

with v_t a stochastic variable with zero mean and constant variance, and where the variable v_t may be interpreted as "news". Equation (D.2) can be rewritten as:

(D.3) $$E_t(X_{t+1}) = X_{t+1} - v_{t+1}\ .$$

Inserting (D.3) in (D.1) yields:

(D.4) $$Y_t = a + b\ X_t + c\ X_{t+1} + w_t$$

with

(D.5) $$w_t = u_t - c\ v_{t+1}\ .$$

All the right hand side variables of equation (D.4) are observable. However, it should be noted that w_t is correlated with X_{t+1}. This implies that equation (D.4) has to be estimated with instrumental variables (or another consistent estimator).

B. A practical solution

We propose the following practical solution. First, we regress:

$$\text{(D.6)} \qquad \ln(X_{t+1}) = h + \sum_{i}^{n} a_i \ln(Zi_t) + s_t$$

with Zi_t, i=1, ..., n a set of instrumental variables, and s_t a stochastic component not correlated with Z. Estimation of (D.6) yields the point estimates $\hat{h}$ and $\hat{a}_i$ for the parameters h and a_i.

Next, we calculate the fitted value of X_{t+1}, i.e.:

$$\text{(D.7)} \qquad \ln(\hat{X}_{t+1}) = \hat{h} + \sum_{i}^{n} \hat{a}_i \ln(Zi_t) .$$

Finally, we insert the fitted value of X_{t+1} in equation (D.4), yielding:

$$\text{(D.8)} \qquad Y_t = a + b\, X_t + c \ln(\hat{X}_{t+1}) + w_t .$$

Since no right hand side variable is correlated with the stochastic component w_t, equation (D.8) can be estimated with ordinary least squares.

Our equations include the expected price of consumption and residential buildings. The set of instruments that we selected to solve the errors in variables problem are lagged prices, indirect taxes, and the output gap. The fitted values were inserted into equations (24) and (25).

IX Appendix E: The Point Estimates: Some Further Details

This appendix shows some more detailed estimation results of the expenditure items reported in the main text.

A. Detailed estimation results

TABLE E1 - Long Run Point Estimates for Private Consumption

	EU	NE	US	JP
cp_l0	-0.54	0.01	-0.45	-0.70
cp_lb	1.14	1.02	1.12	1.08
cp_l2	-0.03	0.09	-0.04	-0.01
cp_l3	-0.01	-0.04	0.03	0.04
-cp_l4	-0.12	-0.56	-0.38	-0.77
Dummies 1972-1981				
Constant	0.08	-0.13	0.00	-0.33
Nominal interest rate	0.03	-0.08	0.00	-0.03
User cost of res. building	0.01	0.04	0.00	-0.06
Real interest rate	0.27	0.53	0.00	1.17
German re-unification	0.00	0.01	0.00	0.00
UK building society	0.00	-0.03	0.00	0.00
Diagnostic statistics				
R2-adjusted	1.00	0.99	1.00	0.99
Durbin Watson	2.16	1.70	1.92	0.95
Log Likelihood	90.73	79.67	86.10	64.40
Dickey Fuller	-5.11	-4.19	-4.84	-2.80
Augmented Dickey Fuller	-4.99	-4.10	-4.74	-2.73

TABLE E2 - Long Run Point Estimates for Money Demand

	EU	NE	US	JP
m_l0	-0.90	-1.61	-1.66	-1.03
m_lb	1.00	1.00	1.00	1.00
m_l2	-0.01	-0.18	-0.04	-0.02
m_l3	0.02	-0.02	-0.07	-0.09
-m_l4	-0.88	-0.57	-2.16	-1.74
Dummies 1972-1981				
Constant	0.06	0.00	0.00	0.00
Nominal interest rate	0.02	0.00	0.00	0.00
User cost of res. building	-0.01	0.00	0.00	0.00
Real interest rate	-0.98	0.00	0.00	0.00
German re-unification	0.04	0.02	0.02	-0.06
UK building society	-0.04	-0.31	0.04	0.00
Diagnostic statistics				
R2-adjusted	0.97	0.96	0.72	0.96
Durbin Watson	1.56	1.45	0.69	1.23
Log Likelihood	68.81	39.28	42.35	52.71
Dickey Fuller	-3.76	-3.61	-2.63	-2.82
Augmented Dickey Fuller	-3.67	-3.53	-2.57	-2.72

TABLE E3 - Short Run Point Estimates for Private Consumption

	EU	NE	US	JP
cp_sb	0.93	0.94	1.01	1.08
	(0.11)	(0.12)	(0.07)	(0.12)
cp_s2	0.01	0.10	-0.02	0.04
	(0.03)	(0.04)	(0.02)	(0.02)
cp_s3	-0.01	-0.02	0.02	-0.03
	(0.01)	(0.02)	(0.01)	(0.02)
cp_s4	-0.40	-0.98	-0.45	-0.47
	(0.39)	(0.27)	(0.21)	(0.43)
cp_sl	-0.91	-0.95	-0.97	-0.52
	(0.26)	(0.31)	(0.22)	(0.18)
cp_s6	0.22	0.09	0.00	0.00
	(0.10)	(0.14)	-.-	-.-
Dummies 1972-1981				
Constant	0.05	-0.33	0.00	-0.05
	(0.11)	(0.13)	-.-	(0.02)
Nominal interest rate	0.00	-0.15	0.00	0.00
	(0.04)	(0.05)	-.-	-.-
User cost of res. building	0.02	0.02	0.00	0.00
	(0.01)	(0.02)	-.-	-.-
Real interest rate	0.11	0.89	0.00	0.39
	(0.47)	(0.33)	-.-	(0.46)
German re-unification	0.00	0.01	0.00	0.00
	(0.01)	(0.01)	-.-	-.-
U.K. Building society	0.00	-0.03	0.00	0.00
	-.-	(0.01)	-.-	-.-
Diagnostic statistics				
R2-adjusted	0.84	0.87	0.84	0.54
Durbin Watson	1.92	1.48	1.45	1.26
Log Likelihood	93.54	87.99	85.13	72.33

TABLE E4 - Short Run Point Estimates for Money Demand

	EU	NE	US	JP
m_sb	1.00	1.00	1.00	1.00
	-.-	-.-	-.-	-.-
m_s2	-0.06	-0.14	-0.16	-0.04
	(0.09)	(0.12)	(0.10)	(0.04)
m_s3	0.03	0.01	0.03	-0.06
	(0.02)	(0.02)	(0.06)	(0.03)
- m_s4	-0.38	-0.43	-1.38	-1.22
	(1.13)	(0.69)	(0.98)	(0.42)
m_sl	-0.80	-0.46	-0.36	-0.62
	(0.31)	(0.26)	(0.17)	(0.24)
Dummies 1972-1981				
Constant	0.13	0.00	0.00	-0.02
	(0.31)	-.-	-.-	(0.03)
Nominal interest rate	0.05	0.00	0.00	0.00
	(0.10)	-.-	-.-	-.-
User cost of res. building	-0.02	0.00	0.00	0.00
	(0.02)	-.-	-.-	-.-
Real interest rate	-1.45	0.00	0.00	0.00
	(1.32)	-.-	-.-	-.-
German re-unification	0.04	0.00	0.00	-0.04
	(0.04)	(0.05)	(0.03)	(0.03)
U.K. Building society	-0.03	-0.21	0.03	0.00
	(0.02)	(0.05)	(0.03)	(0.03)
Diagnostic statistics				
R2-adjusted	0.80	0.50	0.62	0.74
Durbin Watson	1.66	1.44	0.86	1.58
Log Likelihood	67.96	42.36	51.08	55.03

TABLE E5 - Point Estimates for Gross Fixed Capital Formation of Residential Buildings

	EU	NE	US	JP
Short Run elasticities				
Scale	1.09	2.20	4.77	2.86
User cost of res. building	-0.26	-0.69	-0.42	-0.90
Long Run elasticities				
Scale	0.33	0.27	0.27	0.32
User cost of res. building	-0.08	-0.08	-0.02	-0.10
Coefficients				
Adjustment coefficient	0.03	0.19	0.47	0.29
	(0.02)	(0.13)	(0.15)	(0.13)
Rate of depreciation	0.01	0.02	0.03	0.03
	-.-	-.-	-.-	-.-
Constant	0.00	62.63	19.76	2655.83
	-.-	(49.39)	(5.91)	(1260.73)
Scale	2.80	0.78	0.63	0.87
	(0.62)	(0.57)	(0.25)	(0.38)
User cost	-5.92	-10.94	-0.80	-542.55
	(14.54)	(15.52)	(3.91)	(678.75)
Dummies				
German re-unification	0.78	0.00	0.00	0.00
	(1.15)	-.-	-.-	-.-
Constant	0.46	0.00	0.00	0.00
	(0.84)	-.-	-.-	-.-
User cost	12.73	0.00	0.00	0.00
	(22.28)	-.-	-.-	-.-
Diagnostic statistics				
R2-adjusted	0.75	0.70	0.55	0.73
Durbin Watson	1.34	1.03	1.85	1.77
Log Likelihood	60.69	9.11	27.36	-90.61

X References

Barten, A.P. (1984), *Methodological Aspects of Macroeconomic Model Construction*, Cabay, Louvain-La-Neuve.

Bossier, F., Bracke, I., Stockman, P., and Vanhorebeek, F. (2000), "A description of the HERMES II model for Belgium", *Working Paper n°5*, Belgian Federal Planning Bureau.

Bradley, J., Modesto, L., and Sosvilla-Rivero, S. (1995), "HERMIN. A macroeconometric modelling framework for the EU periphery", *Economic Modelling*, Vol. 12, pp. 221-248.

Brayton, F., and Tinsley, P. (eds.) (1996), "A Guide to FRB/US, A Macroeconomic Model of the United States", *Macroeconomic and Quantitative Studies*, Division of Research and Statistics, Federal Reserve Board, Washington, D.C..

Burnside, C. (1998), "Detrending and business cycle facts: A comment", *Journal of Monetary Economics*, Vol. 41, pp. 513-532.

Canova, F. (1998a), "Detrending and business cycle facts", *Journal of Monetary Economics*, Vol. 41, pp. 475-512.

Canova, F. (1998b), "Detrending and business cycle facts: A User's guide", *Journal of Monetary Economics*, Vol. 41, pp. 533-540.

Charemza, W.W., and Deadman, D.F. (1993), *New Directions in Econometric Practice*, Edward Elgar.

Commission of the European Communities (ed.) (1993), *HERMES : Harmonised Econometric Research for Modelling Economic Systems*, North-Holland, Amsterdam.

Cuthberston, K., Hall, S.G., and Taylor, M.P (1992), *Applied Econometric Techniques*, Harvester Wheatsheaf, New York.

Deaton, A., and Muellbauer, J. (1987), *Economics and Consumer Behaviour*, Cambridge University Press, Cambridge.

Engle, R.F., and Granger, C.J.W. (1991), *Long Run Economic Relationships, Readings in Cointegration*, Oxford University Press, Oxford.

Hendry, D.F. (1995), *Dynamic Econometrics*, Oxford University Press.

Johnston, J. (1984), *Econometric Methods*, McGraw-Hill International Book Company.

Laxton, D., Isard, P., Faraqee, H., Prasad, E., and Turtelboom, B. (1998), "MULTIMOD Mark III; the Core Dynamic and Steady-State Models.", *Occasional Paper no. 164*, International Monetary Fund.

Maddala, G.S., and Kim, In-Moo, (1999), *Unit Roots, Cointegration and Structural Change*, Cambridge University Press, Cambridge.

Patinkin, D. (1989), *Money, Interest, and Prices, Abridged*, MIT Press, London.

Powell, A., and Murphy, C. (1997), *Inside a Modern Macroeconometric Model*, Springer-Verlag Berlin.

Roeger, W., and in 't Veld, J. (1997), "QUEST II A Multi Country Business Cycle and Growth Model", *Economic Papers*, European Commission, Directorate-General for Economic and Financial Affairs.